MAT 050

QUANTITATIVE LITERACY LEARNING MODULES

In the following pages of this document you will find a series of learning modules that is designed to address each of the student learning outcomes, competencies, and skills that is listed under the State of Colorado’s Common Course Numbering System (CCNS) for MAT 050 (Quantitative Literacy). Although these learning modules may be used as an online textbook, it is the math departments suggestion that they be used as additional resources to accompany MAT 050’s adopted textbook. Students may need more practice than is included in these modules and further academic support. This work will be free to all of CMC’s instructors and students and will be accessible through CANVAS. This work may be reproduced, but not sold. If you have any questions or comments regarding this document, please don’t hesitate to email me at jvargas@coloradomtn.edu.
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PREFACE

Each MAT 050 learning module begins with a tutorial that is designed to address specific student learning outcomes, competencies, and skills that is listed under the Colorado Community College Common Course Numbering System (CCNS). A link to Khan Academy (khanacademy.org) is given after each tutorial so that students have access to additional learning resources that are specific to the competency being taught. Following the tutorials in each module is a set of homework problems. The homework set begins with a basic set of problems followed by real-world applications, where applicable. “Fun facts” may appear prior to specific applications and should be read prior to completing the problems that follow the fun fact. In most cases, the information given in the “FUN FACT” is necessary to complete the assignment. Finally, at the end of each learning module is a complete set of homework problem solutions. If you find any mistakes in the solutions, or any other part of the learning module, please don’t hesitate to let me know. You can email me at jvargas@coloradomtn.edu. Thank you, and I hope you find these learning modules worthwhile!

-Jason Vargas
COURSE DESCRIPTION:

Develops number sense and critical thinking strategies, introduce algebraic thinking, and connect mathematics to real-world applications. Topics in the course include ratios, proportions, percents, measurement, linear relationships, properties of exponents, polynomials, factoring, and math learning strategies. This course prepares students for Math for Liberal Arts, Statistics, Integrated Math, and college-level career math courses.

STANDARD COMPETENCIES:

I. Demonstrate knowledge of and the ability to solve problems involving ratios, rates, proportions, percents, and measurement conversions.
II. Demonstrate knowledge and usage of formulas.
III. Demonstrate knowledge of and the ability to solve linear equations and inequalities.
IV. Demonstrate knowledge of and the ability to calculate and simplify expressions containing exponents and numeric square roots.
V. Demonstrate knowledge of and the ability to perform algebraic manipulations involving polynomials, polynomial operations, and basic factoring.
VI. Demonstrate the use of critical thinking skills to problem solve.

TOPICAL OUTLINE:

I. Demonstrate knowledge of and the ability to solve problems involving ratios, rates, proportions, percents, and measurement conversions.
   A. Read and write ratios and proportions using colon or fraction form.
   B. Simplify ratios and write rates as unit rates.
   C. Determine whether a proportion is true.
   D. Solve for the missing term of a proportion.
   E. Solve word problems involving proportions.
   F. Convert numbers in percent form to fractional or decimal form and vice versa.
   G. Solve percent problems for base, rate, or amount (percentage).
   H. Solve word problems involving percent using the percent formula or proportions.
   I. Solve percent applications involving topics such as commission, discount, simple interest, and percent increase/decrease.
   J. Identify the basic units in the U.S. system and convert from one unit to another, introducing commonly used fractions as needed.
   K. Reproduce the metric chart (prefixes, abbreviations, and values) from kilo to milli.
   L. Convert from one metric unit to another.
   M. Convert units of length, weight, volume, and temperature between metric and U.S.
systems introducing unit fractions and/or proportions as needed.

II. Demonstrate knowledge and usage of formulas.
   A. Apply formulas in calculating perimeter/circumference and area of plane geometric figures.
   B. Evaluate formulas for given values of the variables, including formulas with integer exponents, fractions, and decimals.
   C. Solve word problems that apply formulas.

III. Demonstrate knowledge of and the ability to solve linear equations and inequalities.
   A. Solve first degree equations including those involving fractions, decimals, ratio, proportion, and percent.
   B. Check the solution of first degree equations.
   C. Graph linear equations in two variables using the Cartesian coordinate system.
   D. Determine x- and y-intercepts of a linear equation.
   E. Find the slope of a line given two points or the equation of the line.
   F. Explain how slope relates to a rate of change in a problem.
   G. Find and write the equation of a line in slope-intercept form.
   H. Solve and graph applications using linear equations.
   I. Solve first degree inequalities, including compound inequalities.
   J. Graph solutions for first degree inequalities.

IV. Demonstrate knowledge of and the ability to calculate and simplify expressions containing exponents and numeric square roots.
   A. Demonstrate proper use of order of operations and properties of exponents, including integer exponents.
   B. Change notation from standard decimal form to scientific notation and vice versa.
   C. Apply properties of exponents to simplify expressions involving scientific notation.
   D. Calculate and simplify square roots of real numbers with both rational and irrational solutions (exact and decimal approximations).

V. Demonstrate knowledge of and the ability to perform algebraic manipulations involving polynomials, polynomial operations, and basic factoring.
   A. Add, subtract, and multiply polynomial expressions with rational coefficients and express the answer in simplest form.
   B. Divide a polynomial by a monomial.
   C. Factor out the greatest common monomial factor.
   D. Factor the difference of two squares.
   E. Factor trinomials of the form $x^2 + bx + c$ (leading coefficient is 1).

VI. Demonstrate the use of critical thinking skills to problem solve.
   A. Model real-world application problems, interpret results, and summarize using complete sentences.
   B. Create and use graphs, tables, and equations to solve real-world application problems relating to linear relationships.
   C. Identify academic support resources.
   D. Engage in appropriate math learning and testing strategies.
   E. Effectively use calculators and other appropriate technology.
Acknowledgements

Colorado Mountain College would like to thank the Morgridge Family Foundation for providing the funding for this project. Please visit morgridgefamilyfoundation.org to learn more about MFF and their mission.
Prerequisite Review

Fractions and decimals with real numbers will be sprinkled throughout these modules and seen in many different MAT 050 course competencies. It would be time well spent completing this review prior to beginning Module I. These problems should be completed without the use of a calculator.

I. Fraction Review

For any problems involving $\pi$, use the approximation $\pi \approx \frac{22}{7}$.

1. Write the following set of numbers in ascending order: \[\left\{ -3\frac{1}{2}, 1\frac{1}{3}, \pi, \frac{0}{10}, -\frac{7}{3} \right\} \]

2. Write the following as mixed numbers in lowest terms:
   - a) $\frac{43}{7}$
   - b) $\frac{102}{20}$

3. Write the following as improper fractions in lowest terms:
   - a) $\frac{4}{6}$
   - b) $-3\frac{2}{5}$
   - c) 27

4. Reduce to lowest terms:
   - a) $\frac{28}{42}$
   - b) $-\frac{12}{32}$

5. Find the LCM of 56 and 20.

6. Evaluate the following:
   - a. $\left(\frac{5}{8}\right)^2$
   - b. $\left(-\frac{1}{2}\right)^4$
7. Perform the indicated operations and simplify:
   \[ a. \frac{5}{21} \cdot \frac{7}{15} \quad b. \quad -3 \frac{1}{5} \cdot 2 \frac{1}{8} \]
   \[ c. \quad -\frac{3}{10} + \frac{8}{15} \quad d. \quad 5 \frac{1}{4} ÷ 7 \frac{1}{8} \]
   \[ e. \quad \frac{2}{9} + \frac{1}{9} \quad f. \quad \frac{4}{27} - \frac{22}{27} \]
   \[ g. \quad \frac{5}{6} + \frac{3}{4} \quad h. \quad \frac{2}{9} - \frac{5}{8} \]
   \[ i. \quad -4 \frac{1}{2} - \left( -3 \frac{1}{4} \right) \quad j. \quad \frac{3x}{4} - \frac{x}{5} \]

8. Simplify the following:
   \[ a. \quad 6 \frac{1}{3} - 4 \left( \frac{7}{12} - \frac{1}{6} \right) \quad b. \quad 8 \left( 3 \frac{1}{2} - \frac{7}{12} \right) ÷ 4 \frac{1}{3} \]
II. Decimal Review

9. Evaluate the following:
   a) \((0.12)^2\)  
   b) \((0.03)^3\)

10. Write the following set of numbers in descending order: \(\{0.33, 0.3\overline{43}, \frac{1}{3}, 0.\overline{343}, 0.331\}\)

11. a) Write “seventy-two and thirty-two ten-thousandths” in decimal notation.
    b) Write 0.068 in word form.
    c) Write 0.068 as a fraction in lowest terms.

12. Write the following as decimal numbers:
    a) \(\frac{3}{16}\)  
    b) \(102\frac{5}{6}\)

13. Perform the indicated operations (do not round):
    a) \(9.9 + 0.377 + 35.6\)
    b) \(28.004 - 16.059\)
    c) \(-2.7 \times 6.85\)
    d) \((-43.212) \div (-7.8)\)
    e) \(49.612 \div (-0.001)\)

14. Evaluate the following and write your answer in decimal form:
    a) \((0.9)^2 - \left(\frac{2}{3} - \frac{3}{10}\right)(6 - 9.5)\)
    b) \((-12.9)^5 + \frac{3}{4}\)

15. True or False. If false, give an example or explain why:
    a) \((0.09)^2 = 0.081\).
    b) \(\frac{4}{7} < 0.5713\).
Solutions to Prerequisite Review:

1. \(-\frac{3}{2}, -\frac{7}{3}, \frac{0}{10}, \frac{1}{3}, \pi\)  
2. a) \(\frac{6}{7}\) b) \(\frac{5}{10}\)  
3. a) \(\frac{14}{3}\) b) \(-\frac{17}{5}\) c) \(\frac{27}{1}\)  
4. a) \(\frac{2}{3}\) b) \(-\frac{3}{8}\)  
5. 280  
6. a) \(\frac{25}{64}\) b) \(\frac{1}{16}\)  
7. a) \(\frac{1}{9}\) b) \(-\frac{34}{5}\) c) \(-\frac{9}{16}\)  
8. a) \(\frac{56}{9}\) b) \(\frac{35}{2}\)  
9. a) 0.0144 b) 0.000027  
10. \(\left\{\frac{33.0}{331.0}, 3\right\}\)  
11. a) 72.0032 b) Sixty-eight thousandths c) \(\frac{17}{250}\)  
12. a) 0.1875 b) 102.8\(\frac{3}{4}\)  
13. a) 45.877 b) 11.945 c) -18.495 d) 5.54 e) -49,612  
14. a) 8.86 b) -10  
15. a) False; \((0.09)^2 = 0.0081\) b) False; \(\frac{4}{7} \approx 0.57142 > 0.5713\)
Module I
Operations with Real Numbers

Because many applications in all fields of study require the use of multiple arithmetic operations (+, -, x, ÷) with signed numbers (numbers that are either positive or negative), we will spend time in this module introducing the real numbers and the basic rules of arithmetic governing them. In addition, we will investigate powers and square roots, order of operations, scientific notation, roman numerals, military time, and then using them to problem solve. We begin by defining the set of real numbers.

I. Real Numbers


To define the set of real numbers we must have a clear understanding of what is meant by the word set. Mathematically speaking, a set is simply a collection of objects. These objects are referred to as the elements of the set. For example, we can define the set of Roaring Fork Campuses to be {Spring Valley Campus, Lappala Center, Glenwood Center}. The three campuses listed in the set are the elements of the set. The use of braces { } around the elements in the set is what we refer to as set-notation. Since there are a countable number (namely three) elements in the set of Roaring Fork Campuses, we say that the set is a finite set. However, many of the sets of numbers that we typically use in mathematics are infinite sets. That is, it would be impossible for us to count the number of elements in an infinite set. Below is a list of some popular infinite sets of numbers:

Number Sets:

- **Natural Numbers**: {1, 2, 3, ...}
- **Whole Numbers**: {0, 1, 2, 3, ...}
- **Integers**: {..., -2, -1, 0, 1, 2, ...}
- **Rational Numbers**: Any number that can be written as a ratio of two integers is a rational number, or a decimal number that either terminates or repeats. That is, the set of rational numbers are all values “x” such that “x” is a ratio of two integers. Translated to set-builder notation: \( \{ x | x \text{ is a ratio of two integers} \} \) (we will revisit set-builder notation in Module VI). Some examples of rational numbers: \( \frac{1}{2}, -\frac{5}{3}, 7, 8.524, 0.575757..., \text{etc.} \)
• **Irrational Numbers**: The set of all numbers that cannot be written as a ratio of two integers. That is, an irrational number is a decimal number that never repeats or never terminates. Some examples of irrational numbers: \( \pi, e, \sqrt{2}, \text{etc.} \). Note: we can approximate irrational numbers, that is, \( \pi \approx 3.14, \ e \approx 2.72, \text{ and } \sqrt{2} \approx 1.41 \)

• **Real Numbers**: Any number that is rational or irrational is a real number.

A **subset** of another set says that every element in the subset is also an element of the set, but the converse is not necessarily true. For instance the set \( A = \{1, 2, 3\} \) is a subset of the set \( B = \{1, 2, 3, 4\} \) since every element in the set \( A \) is also in the set \( B \). \( B \) is not a subset of \( A \) since \( A \) does not contain the element 4. Every set is a subset of itself. You can see from the list above that the natural numbers is a subset of the whole numbers; the whole numbers is a subset of the integers; the integers is a subset of the rational numbers; and the rational numbers is a subset of the real numbers. The following Venn-Diagram illustrates the relationship of real numbers.

II. Ordering Numbers

**Khan Academy Resources**: [https://www.khanacademy.org/math/6th-engage-ny/engage-6th-module-3/6th-module-3-topic-b/v/ordering-rational-numbers](https://www.khanacademy.org/math/6th-engage-ny/engage-6th-module-3/6th-module-3-topic-b/v/ordering-rational-numbers)

When ordering numbers, a **number line** (as illustrated on the next page) is helpful for visualizing the relationship of the numbers that are being ordered. The arrow pointing to the left indicates that the numbers are decreasing to negative infinity “\( -\infty \)”, and the arrow pointing to the right indicates that the numbers are increasing to positive infinity “\( \infty \)”. 

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**MAT 050 – Quantitative Literacy | Operations with Real Numbers**
Inequalities, such as < “is less than” or > “is greater than” also helps us order numbers or algebraic expressions. Reading the statement $a < b$ from left to right we say “$a$ is less than $b$”. We can also read $a < b$ from right to left, in which we say “$b$ is greater than $a$”. $-2 < 3$ is read “negative 2 is less than 3” or equivalently “3 is greater than negative 2”. We will revisit inequalities in greater depth in module VI.

**Example 1:** Replace either < or > for ___ to write a true sentence.

a) $-8___-5$

b) $\frac{23}{15}___\frac{11}{8}$

c) $e___\pi$

**Solutions to Example 1:**

a) **Answer:** $-8 < -5$

b) **Answer:** $\frac{23}{15} > \frac{11}{8}$

c) Since $e \approx 2.72$ and $\pi \approx 3.14$, $e < \pi$. **Answer:** $e < \pi$

### III. Absolute Value and Additive Inverse

**Khan Academy Resources:** [https://www.khanacademy.org/math/arithmetic-home/negative-numbers/abs-value/v/absolute-value-of-integers](https://www.khanacademy.org/math/arithmetic-home/negative-numbers/abs-value/v/absolute-value-of-integers)

The **absolute value** of a number is defined to be the distance that the number is from 0 on a number line. Remember that distances are always positive. That is, the distance from Glenwood Springs to Denver is about 160 miles, and the distance from Denver to Glenwood Springs is still approximately 160 miles. We don’t say the distance is “negative 160 miles” coming home from Denver since it’s going in the opposite direction. So, the expression $|-5|$ is read “the absolute value of -5”, and is defined to be the distance that -5 is from 0 on the number line. So $|-5| = 5$.

**Example 2:** Evaluate the following absolute values.

a) $|-7|$

b) $|32.72|$

c) $|0|$

d) $-|-3|$
Solutions to Example 2:

d) The distance that -7 is from 0 is 7. **Answer: 7**
e) The distance that 32.72 is from 0 is 32.72. **Answer: 32.72**
f) The distance that 0 is from 0 is 0. **Answer: 0**
g) The distance that -3 from 0 is 3. But since there is a “–” sign in front of the absolute value we must apply that to the absolute value, making our answer -3. **Answer: -3**

The **additive inverse** of a number is the amount that we must add to the number to get 0. For example, the additive inverse of 2 is -2 since 2 + (-2) = 0. We can think that the phrase “the additive inverse of” is synonymous with “negative”. We will need to understand additive inverse when describing how to add, subtract, multiply, and divide real numbers.

**Example 3:** Translate each additive inverse sentence to a signed number.

a) The additive inverse of 5.
b) The additive inverse of 0.
c) The additive inverse of -2.
d) The additive inverse of the additive inverse of -6.

Solutions to Example 3:

a) Since 5 + -5 = 0, the additive inverse of 5 is -5. The additive inverse translates to “–”, so the additive inverse of 5 translates to -5. **Answer: -5**
b) Since 0 + 0 = 0, the additive inverse of 0 is 0. **Answer: 0**
c) Since -2 + 2 = 0, the additive inverse of -2 is 2. The additive inverse of -2 translates to – (-2). **Answer: 2**
d) Working backwards, the additive inverse of -6 is 6, and the additive inverse of 6 is -6, so the additive inverse of the additive inverse of -6 is -6. Translation: –(–(-6)). **Answer: -6**

**IV. Addition of Real Numbers**


When adding real numbers it is helpful to think of positive numbers as “assets”, and negative numbers as “debts”. When you add two positive numbers you are adding two assets which will give you a greater asset. When you add two negative numbers you are, in a sense, adding two debts which results a greater debt. When adding a positive number to a negative number you are adding an asset to a debt. The larger absolute value of the two numbers will determine whether the result is an asset or a debt.
Tools for Adding Real Numbers (SUM):
1. If the two numbers have the same sign (either they are both negative or they are both positive) then we may add the numbers absolute value and keep the sign. For example, \((-2) + (-3) = -5\). Think: Adding two debts if they are both negative or two credits, if they are both positive.
2. If the two numbers differ in sign (one is positive and one is negative) then we may subtract the smaller absolute value from the larger absolute value and then keep the sign of the larger absolute value. For example, \((-8) + (3) = -5\). Think: Adding debts and assets. Since the debt has a greater absolute value, the result is a debt (i.e. negative)

Example 4: Add the following real numbers and write your answer in simplest form.

a) \(-8.5 + 4.7\)

b) \(-\frac{11}{18} + \left(-\frac{3}{4}\right)\)

c) \((-31) + 62 + (-8) + (-18)\)

Solution to Example 4:

a) **Answer:** -3.8

b) **Answer:** \(-\frac{49}{36}\)

c) By adding all of the negative numbers we get -57. Now add that amount to 62 to get 5. **Answer:** 5

V. Subtraction of Real Numbers


Tools for Subtracting Real Numbers (DIFFERENCE):
1. To subtract signed numbers it is helpful to rewrite a subtraction statement as an equivalent addition statement. For example 5 – 8 can be written as 5 + (–8). This way we can locate the “assets” and “debts”.
2. When subtracting negative numbers, use the additive inverse to help simplify the statement. For example, 5 – (–8) can be written as an equivalent addition statement 5 + (– (–8)). – (–8) means the additive inverse of -8, which is 8. So we can think that 5 – (–8) = 5 + 8 = 13.

Example 5: Subtract the following real numbers and write your answer in simplest form.

a) 13 – 18

b) -3.2 – 5.8
c) \[- \frac{5}{8} - \left( -\frac{3}{4} \right) \]

d) Subtract -25 from 30.

Solutions to Example 5:

a) \[13 - 18 = 13 + (-18) = -5. \text{ Answer: } -5\]
b) \[-3.2 - 5.8 = (-3.2) + (-5.8) = -9. \text{ Answer: } -9\]
c) \[- \frac{5}{8} - \left( -\frac{3}{4} \right) = \left( -\frac{5}{8} \right) + \left( \frac{3}{4} \right) = -\frac{5}{8} + \frac{6}{8} = \frac{1}{8}. \text{ Answer: } \frac{1}{8}\]
d) “Subtract -25 from 30” translates to \[30 - (-25) = 30 + 25 = 55. \text{ Answer: } 55\]

VI. Multiplication and Division of Real Numbers

Khan Academy Resources:
https://www.khanacademy.org/math/arithmetic-home/negative-numbers/mult-divide-negatives/v/multiplying-negative-real-numbers

https://www.khanacademy.org/math/arithmetic-home/negative-numbers/mult-divide-negatives/v/dividing-positive-and-negative-numbers

Tools for Multiplying (PRODUCT) & Dividing (QUOTIENT) Real Numbers:

1. When multiplying or dividing two numbers that have different signs, then the result will be negative.
2. When multiplying or dividing two numbers that have the same signs, then the result will be positive.
3. When multiplying signed numbers, it is helpful to count the number of negative factors. If there is an even number of negative factors then the result is positive and if there is an odd number of negative factors then the result is negative.
4. Note: WE CANNOT DIVIDE BY 0! For example, \(\frac{5}{0}\) is undefined.

Example 6: Multiply or Divide the following real numbers and write your answer in simplest form.

a) \(-4 \cdot 5\)

b) \(-\frac{32}{-8}\)

c) \((-2)(-1)(-4)(2)(-1)(3)\)

d) \(\frac{3}{4} \cdot -\frac{5}{9}\)
Solution to Example 6:

a) Since we are multiplying a negative number by a positive number, the result is negative.  
   Answer: -20
b) Since we are dividing a negative number by a negative number, the result is positive.  
   Answer: 4
c) Since we are multiplying a series of numbers and there are four negative numbers, the result is positive.  
   Answer: 48
d) Since we are multiplying a positive number by a negative number, the result is negative.  
   \[
   \frac{3}{4} \cdot \frac{5}{9} = -\frac{15}{36} = -\frac{5}{12} \quad \text{Answer: } -\frac{5}{12}
   \]
e) We cannot divide by 0!  
   Answer: Undefined
f) Since we are dividing a negative number by a positive number, the result is negative.  
   \[
   -\frac{3}{5} \div -\frac{7}{10} = -\frac{3 \cdot 10}{5 \cdot 7} = -\frac{30}{35} = -\frac{6}{7} \quad \text{Answer: } -\frac{6}{7}
   \]

VII. Exponents and Square Roots

Khan Academy Resources:

https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-roots/v/introduction-to-square-roots

The number \( 2^3 \) is said to be in **exponential form**. In this example, 2 is the **base** (the number that is repeatedly multiplied), and 3 is the **exponent** (the number of times the base is used as a factor). Note: Any nonzero number raised to a power of zero is equal to 1. Zero to the zero power is undefined. We will investigate rules of exponents in a later Module.

**Example 7:** Evaluate \( 2^3 \).

Solution to Example 7:

\[ 2^3 = 2 \cdot 2 \cdot 2 = 8 \quad \text{Answer: } 8 \]

**Example 8:** Evaluate \((-3)^4\).
Solution to Example 8:
\((-3)^3 = (-3)(-3)(-3) = 81\). Answer: 81

In mathematics, every operation has an inverse operation. For instance, if we raise a number to the second power, say \(4^2\), we have squared the number 4 to give us 16. If we wanted to “undo” what the square did to the number we would take the square root of 16 to get us back to 4.

**Definition of Square Root:** The number \(c\) is a square root of \(a\) if \(c^2 = a\).

Every positive number has two square roots. For instance, the square roots of 25 are 5 and -5, since \(5^2 = 25\) and \((-5)^2 = 25\). The positive square root is called the principal square root. From this point on in this module we will only concern ourselves with the principle (or positive) square root. The symbol \(\sqrt{}\) is called the radical, the number or expression resting underneath the radical is called the radicand. The number \(\sqrt{9}\) literally means, “the principle square root of 9” where 9 is the radicand. For simplicity we read \(\sqrt{9}\) as “the square root of 9”, which equals 3. A perfect square is a whole number that has a square root that is itself a whole number. Examples of perfect squares are 0, 1, 4, 9, 16, 25, 36, 49, etc. Each of these, when we take the square root of them will result in a whole number: \(\sqrt{0} = 0\), \(\sqrt{1} = 1\), \(\sqrt{4} = 2\), \(\sqrt{9} = 3\), \(\sqrt{16} = 4\), etc. The square root of a negative number is not a real number since there does not exist a real number that, when squared, results in a negative number. We will need to know how to work with square roots when solving many real world problems.

**Example 9:** Evaluate the following.

a) \(\sqrt{121}\)

b) \(\sqrt{-64}\)

Solutions to Example 9:

a) Since \(11^2 = 121\), \(\sqrt{121} = 11\). Answer: 11

b) Since there does not exist a real number such that when squared equals "-64", there is no real answer. Answer: No real answer

**Example 10:** Use a calculator to approximate the following. Round your answers to the nearest hundredth.

a) \(\sqrt{97}\)
Solutions to Example 10:

a) \( \sqrt{97} \approx 9.848857802 \). Answer: 9.85

b) \( \sqrt{48} \approx 0.934198733 \). Answer: 0.93

VIII. Order of Operations


If we were asked to simplify \( 8 \div 2 \cdot 4 \), one student may argue that the answer is 1 since \( 2 \cdot 4 = 8 \) and \( 8 \div 8 = 1 \). Another student may argue that the answer is 16 since \( 8 \div 2 = 4 \) and \( 4 \cdot 4 = 16 \). Which student is right when both answers seem reasonable? In this section we will define the proper order of operations and look at some examples that require the use of the order of operations so that we can confirm that \( 8 \div 2 \cdot 4 = 16 \).

**Proper Order of Operations:**

1. Perform operations inside Parentheses ( ), Brackets [ ], Braces { }, Absolute Value |
2. Simplify Expressions with Exponents and Square Roots \( \sqrt{} \)
3. Multiplication and Division (reading from left to right)
4. Addition and Subtraction (reading from left to right)
When simplifying problems involving order of operations it is important to keep your work organized and don’t try to perform too many operations at once. Multiple operations in one step will most likely lead to a mistake. Also, if you have a fractional expression (as in example 11c below), simplify the numerators and denominators separately, then divide the results in the end.

**Example 11:** Simplify the following.

a) \(3(-10)^2 - 8 \div \sqrt{16}\)

b) \(|10(-5)| + 1(-1)\)

c) \(\frac{7^2 - (-1)^3}{3 - 2 \cdot 3^2 + 5}\)

**Solutions to Example 11:**

a) \(3(-10)^2 - 8 \div \sqrt{16}\)

\[
= 3(-10)^2 - 8 \div 4 \\
= 3 \cdot 100 - 8 \div 4 \\
= 300 - 8 \div 4 \\
= 300 - 2 \\
= 298 \\
Answer: 298
\]

b) \(|10(-5)| + 1(-1)\)

\[
= |-50| + 1(-1) \\
= 50 + 1(-1) \\
= 50 + (-1) \\
= 49 \\
Answer: 49
\]

c) \(\frac{7^2 - (-1)^3}{3 - 2 \cdot 3^2 + 5}\)

\[
= \frac{49 - (-1)}{3 - 2 \cdot 9 + 5} \\
= \frac{49 - (-1)}{3 - 18 + 5} \\
= \frac{50}{-15 + 5} \\
= \frac{50}{-10}
\]
\[ \frac{50}{-10} = -5 \]

Answer: -5

**When using a calculator for problems involving order of operations use parentheses properly!**

It is very common for a student to make a mistake simply by how the student is entering data into his/her calculator. Calculators are programmed to understand the proper order of operations, so each entry into a calculator is taken very literally. For example, let’s say that we want to simplify the following: \((2 + 5) \cdot 10\). A student may mistakenly type \(2 + 5 \cdot 10\), and the calculator will show an output of 52, when the answer is 70. The calculator understands that when given the expression \(2 + 5 \cdot 10\), multiplication must come before addition, so 5 is multiplied to 10, then that result is added to 2, giving an answer of 52. To avoid this mistake, it is important to use parentheses when inputting values that require order of operations. With parentheses, the calculator will start simplifying what’s inside the parentheses first.

Another common mistake that students make with calculators involves exponents. For example, suppose that you wish to simplify \((-5)^2\). The answer is 25, but many students input this into their calculator and receive an answer of -25. When this happens, the student failed to input the problem with parentheses. Without parentheses the calculator is reading \(-5^2\), which translates to \(-5 \cdot 5\), which is -25. With parentheses, the calculator reads \((-5)(-5)\), which is 25.

**IX. Scientific Notation**

**Khan Academy Resources:** [https://www.khanacademy.org/math/pre-algebra/pre-algebra-exponents-radicals/pre-algebra-scientific-notation/v/scientific-notation-old](https://www.khanacademy.org/math/pre-algebra/pre-algebra-exponents-radicals/pre-algebra-scientific-notation/v/scientific-notation-old)

*Scientific notation* is commonly used in mathematics to represent positive or negative numbers that are extremely large or are extremely close to zero.

Scientific notation can be defined as follows:

If \(D\) is a decimal number, then in scientific notation \(D = a \times 10^n\) where \(1 \leq |a| < 10\) and \(n\) is an integer. What this definition says is that any decimal number can be written as a number between 1 and 10 (including 1, excluding 10) multiplied by some integral power of 10.

**Writing a number from standard form to scientific notation:**

To write a number that is given in standard form to scientific notation, move the decimal point in the given number to a position so that the number is between 1 and 10 (excluding 10) and drop all 0’s after the last (or before the first) nonzero digit. Then consider how many places and in what direction the decimal point was moved;

a) A decimal movement \(n\) places to the left means that we write the decimal number (between 1 and 10) multiplied by 10 to the power of \(n\) \((n \geq 0)\).
b) A decimal movement \(n\) places to the right means that we write the decimal number (between 1 and 10) multiplied by 10 to the power of \(-n\) \((n \geq 0)\).

**Example 12:** Write the following in scientific notation.

a) \(52,800,000\)

b) \(0.0000000871\)

**Solution to Example 12:**

a) We are moving the decimal point 7 places to the left between 5 and 2, to get \(5.28 \times 10^7\).

   Answer: \(5.28 \times 10^7\)

b) We are moving the decimal point 8 places to the right between 8 and 7, to get \(8.71 \times 10^{-8}\).

   Answer: \(8.71 \times 10^{-8}\)

**Writing a number from scientific notation to standard form:**
Reverse the steps from converting a number in standard form to the equivalent number in scientific notation. If the exponent is positive, move the decimal point to the right and if the exponent is negative, move the decimal point to the left.

**Word names of numbers:**
Each digit in a number represents a certain place-value and can be written as an integral power of 10. Knowing the place-value will help us write the word name of a number. The list below gives place-values for digits to the left of a decimal point from ones to quintillion (starting from the ones digit increasing vertically downward) with the corresponding integral power of 10:

- One \(10^0\)
- Ten \(10^1\)
- Hundred \(10^2\)
- Thousand \(10^3\)
- Ten-thousand \(10^4\)
- Hundred-thousand \(10^5\)
- Million \(10^6\)
- Ten-million \(10^7\)
- Hundred-million \(10^8\)
- Billion \(10^9\)
- Ten-billion \(10^{10}\)
- Hundred-billion \(10^{11}\)
- Trillion \(10^{12}\)
- Ten-trillion \(10^{13}\)
- Hundred-trillion \(10^{14}\)
Quadrillion \(10^{15}\)
Ten-quadrillion \(10^{16}\)
Hundred-quadrillion \(10^{17}\)
Quintillion \(10^{18}\)

Each digit to the right of the decimal point can also be written as an integral power of 10. The list below gives place-values to the right of a decimal point with the corresponding integral power of 10:

Tenths \(10^{-1}\)
Hundredths \(10^{-2}\)
Thousandths \(10^{-3}\)
Ten-thousandths \(10^{-4}\)
Hundred-thousandths \(10^{-5}\)
Millionths \(10^{-6}\)
Ten-millionths \(10^{-7}\)
Hundred-millionths \(10^{-8}\)
Billionths \(10^{-9}\)
Ten-billionths \(10^{-10}\)
Hundred-billionths \(10^{-11}\)
Trillionths \(10^{-12}\)
Ten-trillionths \(10^{-13}\)
Hundred-trillionths \(10^{-14}\)
Quadrillionths \(10^{-15}\)
Ten-quadrillionths \(10^{-16}\)
Hundred-quadrillionths \(10^{-17}\)
Quintillionths \(10^{-18}\)

Notice that each digit to the right of the decimal point can be written as 10 raised to a negative exponent. We will discuss the concept of negative exponents further in the next Module.

**Example 13:** Write the following in standard form. Then write the word name of the number. \(5.32 \times 10^{11}\)
Solution to Example 13:
Because the exponent is positive 11, the decimal point moves to the right 11 places. We add 0’s after the last nonzero digit giving us 532,000,000,000. The name of this number is five hundred thirty-two billion. Answer: 532,000,000,000; Five hundred thirty-two billion

Example 14: Write the following in standard form. Then write the word name of the number.
4.8 \times 10^{-7}

Solution to Example 14:
Because the exponent is negative 7, the decimal point moves to the left 7 places. We add 0’s between the decimal point and the significant figures 4-8 giving us 0.00000048. The name of this number is forty-eight hundred millionths. Answer: 0.00000048; forty-eight hundred millionths

X. Roman Numerals

Roman numerals are used in the medical field when labeling prescriptions and in graphic design for title pages chapter headings (like in this document), and to indicate the weight or cost per 100 (C), 500 (D), or 1000 (M) sheets of paper. The table below gives the Roman numeral with its value.

<table>
<thead>
<tr>
<th>Roman Numerals Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>I = 1</td>
</tr>
<tr>
<td>V = 5</td>
</tr>
<tr>
<td>X = 10</td>
</tr>
<tr>
<td>L = 50</td>
</tr>
</tbody>
</table>
To convert Roman numerals to Hindu-Arabic numerals (our traditional “numbers”), we are either adding values to, or subtracting values from a larger value.

**Converting Roman Numerals to Hindu-Arabic numerals:**
1. Locate the Roman numeral with the largest value.
2. If a Roman numeral of a smaller value is placed after a Roman numeral of a larger value with no other Roman numerals of a larger value following, then we add the smaller value to the larger value. We can keep adding Roman numerals of smaller value to the larger value to increase its value. For example, \( \text{VII} = 5 + 1 + 1 = 7 \).
3. If a Roman numeral of a smaller value is placed before a Roman numeral of larger value, then we subtract the smaller value from the larger value. For example, \( \text{IX} = 10 – 1 = 9 \). In this example it is easier to write IX then VIII, therefore IX is used for 9. We do not keep adding Roman numerals of smaller value in front of a Roman numeral of larger value to keep decreasing its value. We would not write IIX to represent 8, rather we would write VIII.
4. If a Roman numeral of a smaller value rests between two Roman numerals of larger value, then we add the amount of the remaining Roman numerals to the leading Roman numeral. For example, \( \text{XIX} = (X) + (IX) = (10) + (10 – 1) = 19 \).
5. If a bar is placed over a Roman numeral or numerals, then the amount is multiplied by 1000. For example, \( \text{XIX} \) = (10 + 10 – 1) × 1000 = 19,000.
6. In some cases, lower case Roman numerals are used; however, we use the same procedure to translate to Hindu-Arabic numerals.

**Example 15:** Write the following Roman numerals in Hindu-Arabic form.

a) XI
b) LV

c) XCV
d) cdxxiv
e) \( \overline{XLI} \)

**Solution to Example 15:**

a) X is the largest value. Since II follows X, we are adding to the largest value. \( \text{XII} = 10 + 1 + 1 = 12 \). **Answer: 12**

b) L is the largest value. Since V follows L, we are adding to the largest value. \( \text{LV} = 50 + 5 = 55 \). **Answer: 55**

c) C is the largest value and we have smaller values before and after, we would read XCV as \( (XC) + (V) = (100 – 10) + (5) = 95 \). **Answer: 95**

d) We treat this problem the same, even though we have lower case Roman numerals. d is the largest value, therefore we have cdxxiv = \( (cd) + (xx) + (iv) = (500 – 100) + (10 + 10) + (5 – 1) = 400 + 20 + 4 = 424 \). **Answer: 424**

e) Since this Roman numeral has a “bar” we will multiply the result by 1000. L is the largest value, therefore, \( \text{XLIX} = (XL) + (IX) = (50 – 10) + (10 – 1) = 40 + 9 = 49 \). So multiply 49 by 1000 to get 49,000. **Answer: 49,000**
XI. Military Time

In addition to the use of Roman numerals, some health care facilities use military time for medication orders and documentation. Because each hour in military time is unique and based off of a 24-hour clock, “AM” and “PM” is unnecessary, thus eliminating any possible time confusion. Military time is sometimes referred to as “computer time” since computers use the same method of keeping time.

Conversions In Military Time:
In military time, the time is always expressed using four digits. For example, 1 AM becomes 0100.

1. To convert from traditional time to military time:
   a) If traditional time is AM, drop the “:” and add a “0” in the front (if only three digits are present.
   b) If traditional time is PM, drop the “:” and add “12” to the first two digits only if the first two digits are not “12”

2. To convert from military time to traditional time:
   a) If the first two digits are “00”, add “12” to the first two digits and add “:” with “AM”.
   b) If the first two digits are bigger than “00” but less than “12”, add a “:” after the first two digits with “AM”.
   c) If the first two digits are equal to “12”, add a “:” after the first two digits with “PM”
   d) If the first two digits are greater than “12”, subtract “12” from the first two digits, add a “:” after the first two digits with “PM”

Example 16: Convert 8:57 PM to military time.

Solution to Example 16:
Since the given time is “PM” we drop the colon and add 12 to 8, giving us 2057.
Answer: 2057 hours

Example 17: Convert 0015 hours to traditional time.

Solution to Example 17:
Since the first two digits are “00”, we will add 12 to 00 and add a colon with “AM” giving us 12:15 AM. Answer: 12:15 AM
XII. Problem Solving With Real Numbers

Unfortunately, there isn’t one quick algorithm that can be used for solving all types of word problems. It takes a tremendous amount of time, patience, and practice to be able to become an effective problem solver. The list below is intended as a guide to help us through the problem solving process.

Tools for Problem Solving:
1. Read the problem completely and carefully.
2. Determine what we are asked to solve. This information is usually given in the last sentence of the problem.
3. Plan the attack. In any word problem, key words, phrases, or formulas may be given. It is helpful to underline this information.
4. Draw a picture, make a list, create a chart, simulate the situation, or look for a related example to help organize your thoughts.
5. If possible, use estimation to help determine if your answer is correct.
6. Answer the question and check the results.

Example 18: Justin puts $2000 down on a car, then makes monthly payments of $275 for 5 years (60 months). After 6 years, he sells the car for $5200. If he spent a total of $1500 in routine maintenance and minor repairs, what is his net? Is it a profit or a loss? Note: Net is defined to be all costs subtracted from all revenue. \( N = R - C \). This formula appears in a later module.

Solution to Example 18:
Justin’s Revenue is $5200. Justin’s Costs are \( 2000 + (275 \times 60) + 1500 = 20000 \). Justin’s Net = $5200 – 20000 = $14,800 Answer: Net is -$14,800; It’s a loss.
Homework Set:

In problems 1 – 4, add.

1. 74 + 13
2. -9 + (-6)
3. -35 + 47
4. 43 + (-75)

In problems 5 – 8, subtract.

5. -15 – 18
6. -4 – (-19)
7. 0 – 16
8. 35 – (-10)

In problems 9 – 11, multiply.

9. -4 · 0
10. (-14)(31)
11. (-1)(-32)

In problems 12 – 16, divide.

12. 36 ÷ (-3)
13. -48 ÷ 8
14. \[
\frac{-124}{-4}
\]
15. \[
\frac{27}{0}
\]
16. \[
\frac{0}{-2}
\]

In problems 17 – 20, evaluate.

17. \((-1)^2\)
18. \((-1)^3\)
19. \((-5)^0\)
20. \((-2)^4\)
In problems 21 – 22, answer the questions.

21. Explain why \(-5^4\) and \((-5)^4\) represent different values.
22. Explain why \(-5^3\) and \((-5)^3\) represent the same value.

In problems 23 – 29, evaluate, if possible.

23. \(|-2|\)
24. \(\sqrt{81}\)
25. \(\sqrt{-100}\)
26. \(-|-8|\)
27. \(-\sqrt{49}\)
28. \(\sqrt{0}\)
29. \(|0|\)

In problems 30 – 35, answer the following questions.

30. Subtract \(-17\) from \(-12\).
31. Subtract \(-17\) from the sum of \(-21\) and \(-6\).
32. Multiply \(-5\), \(-3\), \(-8\), and \(11\).
33. Divide \(-132\) by \(-11\).
34. Divide \(-49\) into \(147\).
35. Find the product of \(-5\), \(-9\), and \(11\).

In problems 36 – 43, perform the following operations and simplify.

36. \(-36 + (-18) - 23\)
37. \((-8)(-12) - (13)(-5) + (-39)\)
38. \(-8 ÷ (2 - 2)\)
39. \((-7 + 3)(2(3 - 8) + 5^0)\)
40. \(5 - 3[8(7 - 9) + 3 - 5(4 - 9)]\)
41. \(-|-150|\)
42. \(-3\sqrt{9} + 7\sqrt{0} + (-2)(-4)^0 \sqrt{81}\)
43. \(-\left\{3 - \left[14 + 3(-5 - 3) + (-2)^2 - 4^0\right] - 2\right\}\)
44. \(-8\left[63 + (343 - 72) - 4\right]^0\)
In problems 44 – 48, write each number given in scientific notation in standard form, then write the word name.

44. The speed of light is approximately $3 \times 10^8$ meters per second.
45. A light year (lt-yr) is the distance light travels in 1 year and is approximately $5.76 \times 10^{12}$ miles.
46. The approximate distance to the Andromeda galaxy is $2.14 \times 10^6$ light years.
47. The national debt is about $1.304 \times 10^{13}$.
48. Earth is about $9.292 \times 10^7$ miles from the sun.

In problems 49 – 51, write each number given in scientific notation in standard form.

49. A strand of DNA (deoxyribonucleic acid) is about $1.3 \times 10^{-10}$ cm wide.
50. The length of a typical virus cell is $7.5 \times 10^{-8}$ meters.
51. The length of the diameter of a water molecule is $3 \times 10^{-10}$ meters.

In problems 52 – 58, write each number in scientific notation.

52. There are 2,300,000 blocks of stone in Khufu’s pyramid.
53. The distance from the Sun to Pluto is approximately 3,540,000,000 miles.
54. The nearest star to ours (the Sun) is Proxima Centauri at a distance of about $24,700,000,000,000$ miles.
55. Human DNA consists of $5,300,000,000$ nucleotide pairs.
56. An FM radio station broadcasts at a frequency of $102,300,000$ hertz.
57. The mass of a proton is generally given as $0.000 \ 000 \ 000 \ 000 \ 000 \ 000 \ 000 \ 000 \ 000 \ 001 \ 670$ kg.
58. As of 2006, the smallest microprocessors in common use measured $0.000 \ 000 \ 006 \ 5$ m across.

In problems 59 – 60, answer the following in scientific notation.

59. A gigabyte is a measure of a computer’s storage capacity. One gigabyte holds about one billion bytes of information. If a firm’s computer network contains 2500 gigabytes of memory, how many bytes are in the network?

60. In the United States, 200 million gallons of used motor oil is improperly disposed of each year. One gallon of used oil can contaminate one million gallons of drinking water. How many gallons of drinking water can 200 million gallons of oil contaminate? (Source: The Macmillan Visual Almanac)
1. APPLICATIONS FOR EMT/MEDICAL ASSISTANT/NURSING

FUN FACT: In the medical field, when using the apothecary or household systems of measurement to label containers of medicine, the units of measurements are abbreviated and listed first followed by the dose, written with Roman numerals in lower case. For example, gr v is the correct way to write “5 grains”. Recall: grain (gr), Drams (dr), ounce (oz), minim (m), fluid dram (fl dr).

In problems 61 – 63, write the following measurements using the apothecary symbols and Roman numerals.

61. 3 grains
62. 6 ounces
63. 2 drams

In problems 64 – 67, For the following use your knowledge of the apothecary system and Roman numerals to write each measurement in word form.

64. gr ccxl
65. dr xvi
66. fl dr xxiv
67. m xv

In problems 68 – 70, convert the following traditional time to military time.

68. 1:55 AM
69. 11:40 AM
70. 7:32 PM

In problems 71 – 73, convert the following military time to traditional time.

71. 0812 hours
72. 2325 hours
73. 1234 hours

In problems 74 – 81, answer the following questions.

74. A prescription indicates that a patient is to receive 10 milligrams of aminophylline per kilograms that the person weighs. If the person weighs 74 kilograms, then how many milligrams should the patient receive?
75. A nurse sets the drip rate for an IV at 26 drops each minute. How many drops does the patient receive in 1 hour?
76. On average, the human heart beats 70 times in 1 minute. How many beats is this in a year? (Use 365 days in a year.) If the average male in the United States lives to be 74, estimate
the number of heartbeats in an average male’s lifetime. If the average female life span in the United States is about 80 years, estimate the number of heartbeats in the average female’s lifetime.

77. Carl is to set an IV drip so that a patient receives 840 milliliters of 5% D/W solution in an hour. How many milliliters should the patient receive each minute?

78. A patient is to receive radiation treatment for cancer. The treatment calls for a total of 324 rads administered in 8 bursts of focused radiation. How many rads should each burst be?

79. The FDA recommends that for a healthy diet a person should limit fat intake to about 65 grams of fat per day. If a person splits the recommended fat intake equally among three meals, how much fat is allowed in each meal?

80. A patient with a mass of 55 kilograms is to receive an antibiotic. The order is to administer 9.5 milligrams per kilogram. How much of the antibiotic should be given?

81. A patient weighing 132 pounds is to receive 0.04 milligram per kilogram of clonazepam (an anticonvulsant drug). If each tablet contains 0.5 milligrams, how many tablets should the patient receive?

2. APPLICATIONS FOR FIRE SCIENCE

FUN FACT: Midflame windspeed (MFWS) is defined as the velocity of the winds, in miles per hour (mi/hr), taken at the mid-height of the flames. MFWS will directly affect the direction of movement of the flaming front and is important in fire spread calculations. The midflame windspeed is determined by use of the wind adjustment table (Figure 3.1), which provides values in terms of fuel exposure and fuel model.

![Figure 1.1 Adjustment factors for midflame wind.](image-url)
The adjustment values are typically applied to the 20-foot windspeed, which is the speed that is measured 20 feet above any fuel or obstruction, usually by a weather station. The midflame windspeed is obtained by multiplying the 20-foot windspeed by the appropriate wind adjustment factor from the table (Figure 3.1).

**Figure 1.2 Effects of fuel sheltering on wind speeds**

It is very important to know which fuel model and sheltering configuration is being studied, and whether a given windspeed is a 20-ft windspeed or an already adjusted midflame windspeed.

**Figure 1.3 Typical fuel sheltering for slope locations.**
The midflame windspeed will be LESS THAN the 20-foot windspeed, because vegetation and friction slow down winds closer to the surface. That is why all the adjustment factors in the table are less than 1.

To calculate the midflame windspeed, multiply the 20-foot windspeed by the correct adjustment factor given on figure 3.1. In problems 82 – 84, use the correct adjustment factor in Figure 3.1 to answer the following questions.

82. A fire is burning in a fully sheltered area of dense, or closed, stands described as Fuel Model 4. The local weather station reports the 20-ft windspeed is 15 miles per hour. What is the midflame windspeed?
83. A 20-foot windspeed at the top of the ridge is reported to be 35 miles per hour, with fuel model 11 vegetation. What is the midflame windspeed?
84. A 20-foot windspeed in unsheltered fuels at the base of the ridge is reported to be 18 miles per hour. The vegetation is fuel model 13. Determine the midflame windspeed.

Note: To calculate resultant forces add upward and downward forces. Upward forces are positive and downward forces are negative. In problems 85 – 86, answer the following questions regarding resultant force.

85. An elevator weighs 745 pounds and the steel cable connected to the top adds another 300 pounds. Three people enter the elevator. One person weighs 145 pounds, the second weighs 185 pounds, and the third 168 pounds. The motor exerts an upward force of 1800 pounds. What is the resultant force?
86. A 500 pound concrete block is suspended by two cables. Each cable is exerting 250 pounds of upward force. What is the resultant force? What does this tell you?

3. APPLICATIONS FOR ACCOUNTING

In problems 87 – 96, answer the following questions.

87. Suppose you have $452 in savings. Following is a list of your deposits and withdrawals for the month of July. What is your balance at the end of July?

<table>
<thead>
<tr>
<th>Deposits</th>
<th>Withdrawals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$45</td>
<td>$220</td>
</tr>
<tr>
<td>$98</td>
<td>$25</td>
</tr>
<tr>
<td>$88</td>
<td>$10</td>
</tr>
<tr>
<td>$54</td>
<td></td>
</tr>
</tbody>
</table>
88. An accountant is given the following spreadsheet of expenses and income for a company. What is the final balance?

<table>
<thead>
<tr>
<th>Description of Expense</th>
<th>Amount</th>
<th>Description of assets/income</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payroll</td>
<td>$16,980</td>
<td>Checking Account</td>
<td>$2,359</td>
</tr>
<tr>
<td>Utilities</td>
<td>$1,250</td>
<td>Income</td>
<td>$41,300</td>
</tr>
<tr>
<td>Water</td>
<td>$158</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Waste Disposal</td>
<td>$97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>New Inventory</td>
<td>$12,341</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

89. Suppose you have a current balance of -$1243 on your credit card at the end of April. During the month of May, you make the transactions shown in the table below. What is your balance at the end of May?

Charges:
- Truman’s $58
- Dave’s Diner $13
- Fuel’n Go $15
- Finance Charge $18
- Payment: $150

90. Suppose the following is your checkbook register. Find your final balance.

<table>
<thead>
<tr>
<th>Date</th>
<th>No.</th>
<th>Transaction Description</th>
<th>Debits</th>
<th>Credits</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-24</td>
<td></td>
<td>Deposit</td>
<td></td>
<td>428</td>
<td>682</td>
</tr>
<tr>
<td></td>
<td>952</td>
<td>Food Lion</td>
<td>34</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>2-25</td>
<td>953</td>
<td>Cash</td>
<td>30</td>
<td>00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>954</td>
<td>Green Hills Apts.</td>
<td>560</td>
<td>00</td>
<td></td>
</tr>
<tr>
<td>2-25</td>
<td>955</td>
<td>Electric &amp; Gas</td>
<td>58</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>2-26</td>
<td>956</td>
<td>American Express</td>
<td>45</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>

91. Suppose your account balance was $172. If you deposited a $27 check, bought a $156 money order, and withdrew $25 cash, what is your new balance?

92. Suppose that your checking account balance was $349. During lunch you decide to deposit your paycheck of $429 and pay a $72 phone bill. What is your new balance?
93. Suppose that you are raising funds for charity on Celebrity Apprentice. Currently, you have raised $280,540 and you intend to add $28,580 of your own money. Your competitor has raised a total of $485,300. How much more do you need to raise to match your competitor’s funds?

94. Suppose that your gross annual salary is $34,248. If you get paid on a monthly basis, what is your gross monthly salary?

95. As a financial planner you are asked by a client to split $16,800 evenly among 7 investments. How much do you put into each investment?

96. The federal government grants a state $5,473,000 to be distributed equally among the 13 technical colleges in the state. How much does each technical college receive?

In problems 97 – 98, use the table below to answer the questions.

<table>
<thead>
<tr>
<th>Reimbursements for Travel on Business</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.35 per mile when using own vehicle</td>
</tr>
<tr>
<td>$0.15 per mile when using company vehicle</td>
</tr>
<tr>
<td>$10 breakfast when traveling between 12 A.M. and 11 A.M.</td>
</tr>
<tr>
<td>$12 lunch when traveling between 11 A.M. and 3 P.M.</td>
</tr>
<tr>
<td>$15 dinner when traveling between 3 P.M. and 12 A.M.</td>
</tr>
</tbody>
</table>

97. Suppose you work for a company that reimburses travel at the rates given in the table above. On a business trip, you use your own vehicle, leave on Tuesday at 11:30 A.M. and note that the odometer reads 75,618.4. You return on Thursday and arrives at 6:45 P.M. At the conclusion of your trip, you find that the odometer reads 76,264.1. How much should she be reimbursed for mileage and food from your company?

98. Suppose you work for a company that reimburses travel at the rates given in the table above. On a business trip, you use a company vehicle and leave at 7:00 A.M. on a Monday. You note that the odometer reads 45,981.6. You return on Friday, arriving at 2:15 P.M. At the conclusion of the trip you note that the odometer reads 46,610.8. How much should you be reimbursed for mileage and food?

99. Jason’s checking account shows a balance of $24. Unfortunately, he forgot about a check for $40 and it clears. The bank then charges $17 for insufficient funds. What is his new balance?

100. Suppose that your closing credit card balance in January was -$1320. In February, you make a payment of $450 and another of $700. Your finance charges were $19. You then make purchases in the amounts of $32 and $27. What is your February closing balance?

101. Suppose that you have a balance of $126 in your bank account. A check written against your account for $245 arrives at the bank.
   a) What is your balance?
   b) Because you had insufficient funds to cover the check amount, the bank assesses a service charge of $20. What is your balance after the service charge?
102. Suppose that you a balance of -$37 in your bank account. To avoid further charges, you must have a balance of $30. What is the minimum amount that you can deposit to avoid further charges?

In problems 103 – 107, use the equation Net = Revenue – Cost to answer the following questions. If the net is positive it is a profit. If the net is negative, it is a loss.

103. The revenue for a company in 2010 was $3,568,250 and their total costs were $1,345,680. What was the net? Was it a profit or loss?
104. Suppose a client owns a business that is in financial trouble. You inform your client that his net worth currently is -$5267. To stay in business, his net worth should be at least $2500 by the end of the month. What is the minimum profit he must earn in order to remain in business?
105. Suppose you put a down payment of $800 to purchase a vehicle. You make 60 payments of $288 and spend $950 in maintenance and repairs. Three years after paying off the car you sell it for $4000. What is your net? Is it a profit or loss?
106. In 1959, Jason put $800 down on a new Chevy Impala. He made 24 payments of $18 and in 2000, he spends $2500 restoring the car for a classic car show. At the show, someone offers him $20,000 for the car. If he paid approximately $3000 in maintenance over the years, what would be his net if he accepted the offer? Is it a profit or loss?
107. The table below lists your 2010 assets and debts. Calculate your net worth in 2010.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Debts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings = $1498</td>
<td>Credit Card Balance = $1841</td>
</tr>
<tr>
<td>Checking = $2148</td>
<td>Mortgage = $74,614</td>
</tr>
<tr>
<td>Furniture = $18,901</td>
<td>Automobile = $5488</td>
</tr>
<tr>
<td>Jewelry = $3845</td>
<td></td>
</tr>
</tbody>
</table>

4. APPLICATIONS FOR CULINARY ARTS

FUN FACT: A restaurant owner may calculate the contribution margin to help determine the menu price for a specific food item. The contribution margin can be calculated by subtracting the cost of the food from the selling price.

In problems 108 – 111, use the given information and the equation Contribution margin = selling price – food cost to calculate the contribution margin.

<table>
<thead>
<tr>
<th>Item</th>
<th>Food Cost</th>
<th>Selling Price</th>
<th>Contribution Margin</th>
</tr>
</thead>
<tbody>
<tr>
<td>108. A</td>
<td>$3.00</td>
<td>$10.00</td>
<td>$________</td>
</tr>
<tr>
<td>109. B</td>
<td>$4.00</td>
<td>$12.00</td>
<td>$________</td>
</tr>
</tbody>
</table>
In problems 110 – 111, use the given information and the equation Contribution margin = selling price – food cost to calculate the contribution margin.

<table>
<thead>
<tr>
<th>Item</th>
<th>Selling Price</th>
<th>Food Cost</th>
<th>Contribution Margin</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>C $5.00</td>
<td>$15.00</td>
<td>$________</td>
</tr>
<tr>
<td>111</td>
<td>D $6.00</td>
<td>$17.00</td>
<td>$________</td>
</tr>
</tbody>
</table>

In problems 112 – 116, use the given information and the equation Contribution margin = selling price – food cost to calculate the contribution margin.

<table>
<thead>
<tr>
<th>Item</th>
<th>Selling Price</th>
<th>Food Cost</th>
<th>Contribution Margin</th>
</tr>
</thead>
<tbody>
<tr>
<td>112. Fish</td>
<td>$21.00</td>
<td>$_______</td>
<td>$12.50</td>
</tr>
<tr>
<td>113. Beef</td>
<td>$_______</td>
<td>$12.00</td>
<td>$13.00</td>
</tr>
<tr>
<td>114. Chicken</td>
<td>$18.50</td>
<td>$4.50</td>
<td>$ ________</td>
</tr>
<tr>
<td>115. Veggie</td>
<td>$17.50</td>
<td>$_______</td>
<td>$13.50</td>
</tr>
<tr>
<td>116. Game</td>
<td>$_______</td>
<td>$13.00</td>
<td>$15.00</td>
</tr>
</tbody>
</table>

In problem 117, use the information below to calculate the value of each item in the meat inventory and the total. Round each value to the nearest cent.

<table>
<thead>
<tr>
<th>Item</th>
<th>Purchase Price</th>
<th>Amount in Inventory</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Prime Rib</td>
<td>$6.69 per #</td>
<td>#120.3</td>
<td>$______</td>
</tr>
<tr>
<td>b) Tenderloin</td>
<td>$12.05 per #</td>
<td>#187.7</td>
<td>$______</td>
</tr>
<tr>
<td>c) N.Y. Strips</td>
<td>$9.56 per #</td>
<td>#46.4</td>
<td>$______</td>
</tr>
<tr>
<td>d) Meat Inventory Value</td>
<td>TOTAL $_______</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In problems 118 a – j, a restaurant owner wants to determine which would be more profitable: to make or buy Cheesecake. Perform a “Make or Buy Analysis” by answering the questions using the given data.

118. Frozen Cheesecake – 6 cakes per case, 12 slices per cake - $58.00 per case

To Make 4 cakes, 10 slices per cake:
Ingredient Cost $14.00
Total Time required: 1 hour and 25 minutes
Rate of pay - $11.00 per hour

Determine the End Product Prime Cost per slice for the bought and made products.

Make or Buy Analysis

<table>
<thead>
<tr>
<th>Question</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>Production Time required (in minutes)</td>
</tr>
<tr>
<td>b)</td>
<td>Hourly Rate of employee</td>
</tr>
<tr>
<td>c)</td>
<td>Actual hourly labor cost (hourly rate x 1.25)</td>
</tr>
<tr>
<td>d)</td>
<td>Cost per minute (actual hourly cost / 60)</td>
</tr>
<tr>
<td>e)</td>
<td>Total labor cost (number of minutes x cost per minute)</td>
</tr>
</tbody>
</table>
For Recipes:

f) Total Food cost for “Recipe” $_______.____
g) End Product Prime Cost (Total labor cost + Total Food cost) $_______.____
h) E.P.P. Cost per Unit (E.P.P. Cost / number of units produced)
   Units may be in gallons, pounds, ounces, each, etc. $_______.____

Analysis:

i) Cost per Unit to Buy prepared Item $_______.____
j) Which is less expensive?

FUN FACT: A yield test produces valuable information about the actual as-purchased (AP) cost of meat and poultry products that are fabricated within the foodservice operation.

In problem 119, fill in the missing values for the following “Butcher’s Yield Test”.

119.

**BUTCHER’S YIELD TEST**

Item Tested: Chicken Breast
Tested By: ___________________________ Date of Test: July 14, 2010
Intended Use of Product: 6 oz. Chicken Marsala
Purveyor: Nobel/Sysco

AP Price per Pound: $3.59
AP Weight: 19 # 3 oz.

AP Cost = $_______.____ = (AP price per pound) × (A.P. Weight)

<table>
<thead>
<tr>
<th>Trim Item</th>
<th>Trim Wt.</th>
<th>Value per #</th>
<th>= Trim Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bones</td>
<td>2# 4 oz.</td>
<td>$.79</td>
<td>$___<strong><strong>.</strong></strong></td>
</tr>
<tr>
<td>Fat and Skin</td>
<td>3# 6 oz.</td>
<td>$.29</td>
<td>$___<strong><strong>.</strong></strong></td>
</tr>
</tbody>
</table>

Total Trim Weight $_______.____ Total Trim Value $_______.____

New Fabricated Weight (NFW) a.k.a. E.P. weight = ________ = (A.P. weight) – (Total Trim weight)

Yield % = ________ % = (New Fabricated Weight) ÷ (A.P. weight)

Adjusted Fabricated Cost (AFC) = ________ = (A.P. Cost) – (Trim Value)

Cost Factor per pound = ________ = (AFC ÷ NFW)

Cost per portion = $_______.____

New Adjusted Price per # = (New A.P. cost per #) × (Cost Factor)
$_______.____ = ($3.50) × (Cost Factor)

New Adjusted Price per portion = $_______.____ (New Adjusted Fabricated Cost per #) × (portion weight in oz. / 16)
5. APPLICATIONS FOR EARLY CHILDHOOD EDUCATION

In problems 120 – 123, write the word name of each number.

120. 48,921,138
121. 10,284,420,239,132
122. 1,238,000,000,000,000
123. 67,000,000,000,000,000,000

FUN FACT: We can determine quickly, without a calculator, whether a given number is divisible by the numbers 2, 3, 4, 5, 6, 9, or 10 by using divisibility rules. Below are the following divisibility rules for 2, 3, 4, 5, 6, 9, and 10. There are interesting divisibility rules for the numbers 7 and 11, but we will not discuss them here. Ask your instructor to see if he/she knows the divisibility rules for 7 and 11.

**Divisibility Rules:**
- 2: An integer is divisible by 2 if the number is even.
- 3: An integer is divisible by 3 if the sum of the digits in the number is divisible by 3. For example, 765 is divisible by 3 since $7 + 6 + 5 = 18$ and 18 is divisible by 3.
- 4: An integer is divisible by 4 if the last two digits of the number is divisible by 4. For example, 8724 is divisible by 4 since 24 is divisible by 4.
- 5: An integer is divisible by 5 if the last digit of the number is either 0 or 5.
- 6: An integer is divisible by 6 if the number is divisible by both 2 and 3.
- 9: An integer is divisible by 9 if the sum of the digits in the number is divisible by 9.

In problems 124 – 127, use the divisibility rules to determine whether the given number is divisible by 2, 3, 4, 5, 6, 9, or 10.

124. 885
125. 9528
126. 10,800
127. 875,255,212

FUN FACT: A number is **prime** if it is a natural number greater than 1 and is only divisible by 1 and itself. A number is **composite** if it is a natural number greater than 1 and is divisible by any number other than 1 and itself. The numbers 0 and 1 are neither prime nor composite.

In problems 128 – 133, determine whether each number is prime, composite or neither. You may want to use the divisibility rules given above to help determine your answer.

128. -7
129. 2
130. 87
131. 131
132. 413
133. 547

FUN FACT: A natural number that divides evenly into another natural number is called a factor of the number. For example, 6 is a factor of 30 since 6 divides evenly into 30.

In problems 134 – 141, determine ALL of the factors of the given numbers. You may want to use the divisibility rules given above to help determine your answer.

134. 34
135. 9
136. 72
137. 94
138. 69
139. 96
140. 23
141. 82

In problems 142 – 147, convert the Roman numerals to Hindu-Arabic numerals.

142. CXII
143. CLXXXI
144. DCIII
145. CCCLXXXVIII
146. CMLXXXVII
147. DCCXIV

FUN FACT: Statistical measures of center such as mean, median and mode are being taught as early as elementary school. The mean is the arithmetic average of a given set of data. The mean can be found by adding the values of all of the data, then dividing this amount by the number of data values. The median is the middle most number of a set of organized data (either the data is in ascending order or in descending order). If there is an odd number of organized data, the median is the middle number, if there is an even number of organized data then the median is the average of the two middle numbers. The mode is the data value that occurs at the highest frequency. If no data value is listed more than once, then the data has no mode. If two data values “tie” for the mode, then the set has two modes and is called bimodal. If more than two data values “tie” for the mode, then the set has that many modes and is called multimodal.

In problems 148 – 151, find the mean, median, and mode of the given set of data values.

148. 4, 32, 23, 48, 17, 39, 28, 5
149. 21, 10, 22, 47, 39, 9, 43
150. 17, 5, 4, 27, 5, 42, 4, 23, 39, 26
151. 39, 13, 17, 31, 10, 40, 46, 14, 49, 14, 7
6. APPLICATIONS FOR GRAPHIC DESIGN/PROFESSIONAL PHOTOGRAPHY

In problems 152 – 154, answer the following questions.

152. A printing company is asked to make 800 business cards for a client. It can print 12 cards on each sheet of card stock. How many sheets will be used?
153. An employee of a copying company needs to make small fliers for a client. He can make 4 fliers out of each piece of paper. The client needs 500 fliers. How many pieces of paper must be used?
154. An employee of a copying company needs to make leaflets for a client. She can make 6 leaflets out of each piece of paper. The client needs 1800 leaflets. How many pieces of paper must be used?

FUN FACT: In graphic design, envelopes are available in a variety of sizes. The sizes are measured in two dimensions (length and width) in inches. Typically, the dimensions of the suggested enclosure will be no less than ¼ inch difference between the dimensions of the envelope, however, due to the many different types of envelopes there are exceptions to this rule as seen in the following problems.

In problems 155 – 158, answer the following questions.

155. A commercial envelope with dimensions 3.875” by 7.5” suggests that the enclosures dimensions be \( \frac{1}{8} \)” by a quarter inch less than the dimensions of the envelope. What should the dimensions of the enclosure be?
156. A square envelope with length 9.5” suggests that the enclosures dimensions be ¼” less than the dimensions of the envelope. What should the dimensions of the enclosure be?
157. An enclosure measures 8.5” × 10.75”. If the envelopes dimension should be increased by 0.25” × 0.5” to fit the enclosure, what should be the dimensions of the envelope?
158. An enclosure measures 5.25” × 7.625”. If the envelopes dimension should be increased by 0.25” × 0.5” to fit the enclosure, what should be the dimensions of the envelope?

In problems 159 – 160, answer the following questions.

159. Josh found that there were \( 16 \frac{3}{4} \) reams of paper in a storage cabinet. He needed to use \( 5 \frac{5}{8} \) reams for one job and \( 1 \frac{2}{3} \) reams for another job. How much paper was left after Josh completed the jobs?
160. A litho press operator was responsible for running three jobs during a standard 7 ½-hour workday. One job took 3 hours 25 minutes, and the second job took 2 hours 15 minutes. How many hours were left to complete the 3rd job?
In problems 161 – 162, use the table above to help convert and write your answer in scientific notation.

161. Convert 7 petabytes to megabytes.
162. Convert 800 gigabytes to zettabytes.

FUN FACT: Photographers may need to convert the dimensions of the “actual size” of a file, given in inches, to pixel dimensions using ppi (pixels per inch). Knowing the pixel dimension will help photographers determine input resolution based off of what they wish their image output to be. For example, a 2” × 3” file at 900 ppi has pixel dimensions of (2 × 900) by (3 × 900) = 1800 by 2700. If we wanted to determine the maximum size print on a 300 ppi printer, we would divide 300 into 1800 and 2700, giving us 6 × 9 inches.

In problems 163 – 166, convert the actual size of the file to pixel dimensions.

163. If you have a file that is 6 × 9 inches at 300 ppi, what are the pixel dimensions?
164. If you scan a 4 × 6 inch image in at 900 ppi, what is the maximum size print you can output on a 300 ppi printer?
165. If you have an image that is 2 × 2.5 inches and you need to output it at 16 × 20 at 300 ppi, what should your input resolution be?
166. If you have a 35mm negative (1 × 1.5 inches) and you need to output it at 9 × 13.5 at 300ppi, what should your input resolution be?

7. APPLICATIONS FOR INTEGRATED ENERGY TECHNOLOGY

In problems 167 – 172, answer the following questions.

167. You wish to purchase a solar PV (photovoltaic) system for your home. The cost for a solar PV system is calculated from the array size given in kilowatts. If you need to purchase a solar PV system with 6.7 kW array size at $8/watt, how much will you pay for your solar PV system?
168. You wish to purchase a solar PV (photovoltaic) system for your home. The cost for a solar PV system is calculated from the array size given in kilowatts. If you need to purchase a solar PV system with 4.3 kW array size at $9.50/watt, how much will you pay for your solar PV system?

169. You purchased a solar PV (photovoltaic) system for your home. You paid $72,000 for the system at $10.99/watt. What array size, in kilowatts, did you purchase?

170. You purchased a solar PV (photovoltaic) system for your home. You paid $42,000 for the system at $7.49/watt. What array size, in kilowatts, did you purchase?

171. You purchased a solar PV (photovoltaic) system for your home. You paid $56,234 for an array size of 7.2 kW. How much did you pay per watt?

172. You purchased a solar PV (photovoltaic) system for your home. You paid $67,890 for an array size of 8.1 kW. How much did you pay per watt?

8. APPLICATIONS FOR PROCESS TECHNOLOGY

In problems 173 – 177, write each number given in scientific notation in standard form.

173. 1 foot-pound force is approximately $1.286 \times 10^{-3}$ British thermal units. Write $1.286 \times 10^{-3}$ in standard form.

174. 1 foot-pound force is approximately $3.766 \times 10^{-7}$ kilowatt-hours. Write $3.766 \times 10^{-7}$ in standard form.

175. 1 British thermal unit is approximately $2.928 \times 10^{-4}$ kilowatt-hours. Write $2.928 \times 10^{-4}$ in standard form.

176. 1 kilowatt-hour is approximately $3.6 \times 10^6$ Joules. Write $3.6 \times 10^6$ in standard form.

177. 1 Terawatt-year is approximately $8.76 \times 10^{12}$ kilowatt-hours. Write $8.76 \times 10^{12}$ in standard form.

In problems 178 – 183, write each number in scientific notation.

178. 1 kilowatt-hour is approximately 3414 British thermal units. Write 3414 in scientific notation.

179. 1 Exajoule is one quintillion Joules (1,000,000,000,000,000,000). Write one quintillion Joules in scientific notation.

180. Write 1 quadrillion British thermal units in scientific notation.

181. 1 therm is 100,000 British thermal units. Write 100,000 British thermal units in scientific notation.

182. 1 barrel of oil is approximately equivalent to 5,800,000 British thermal units of energy. Write 5,800,000 British thermal units in scientific notation.

183. 1 gallon per minute is approximately 0.002228 cubic feet per second. Write 0.002228 in scientific notation.
In problems 184 – 185, answer the following questions.

184. If in 2011 the U.S. produces 7,513,000 barrels of petroleum products per day and consumes 19,148,000 barrels of petroleum products per day. If 1 barrel of petroleum products is 42 gallons, how many gallons of petroleum products per day must the U.S. import from other countries to meet the daily demand?

185. Suppose that in 2011 47.2% of all of the U.S. petroleum consumption is used for motor gasoline. If the U.S. consumes a total of 19,148,000 barrels per day of petroleum products, and there are 42 gallons in one barrel, and the average cost per gallon of motor gasoline is $3.52, how much money is spent per day in the U.S. for motor gasoline?

FOR PROBLEMS 186 – 188: Recall from Module I, page 28, the “FUN FACT” for finding the hydraulic gradient. In order for us to determine the direction of water flow we need to consider three wells and a line between the two wells that have the highest and lowest heads (see figure 2 on page 29). In the following problems you will be given the head for each of three wells and the distance between each well. You will need to divide the difference between the head of the lowest well and the head of the highest well into 0.1 increments, then determine what each increment represents as a distance from each well. I provide an example below for your consideration.

Practice Example: Well A = 10.4 meters head, Well B = 10.0 meters head, and Well C = 9.9 meters head in the given figure below form a right angle (i.e. 90° angle). We will discuss the importance of a right angle in module VI. The distance between well A and well B is 500 meters and the distance between well A and well C is 500 meters. Now, divide the line between the two wells with the highest and lowest head into equal parts of 0.1 meters head, then determine what distance each increment of 0.1 meters head represents.

Solution: Since well A has the highest head (10.4 m) and well C has the lowest head (9.9 m) we will only consider these two wells to partition in increments of 0.1 m. Then since there are five increments of 0.1, we take 500 meters and divide by 5, giving us 100 m per 0.1 m increment.
In problems 186 – 188, answer the following questions.

186. Well A = 15.4 meters head, Well B = 9.0 meters head, and Well C = 9.8 meters head in the given figure below. The distance between well A and well B is 300 meters and the distance between well A and well C is 250 meters. Divide the line between the two wells with the highest and lowest head into equal parts of 0.1 meters head, then determine what distance each increment of 0.1 meters head represents.

![Diagram](image)

187. Well A = 13.7 meters head, Well B = 12.2 meters head, and Well C = 10.8 meters head in the given figure below. The distance between well A and well B is 570 meters and the distance between well A and well C is 550 meters. Divide the line between the two wells with the highest and lowest head into equal parts of 0.1 meters head, then determine what distance each increment of 0.1 meters head represents.

![Diagram](image)

188. Well A = 8.9 meters head, Well B = 8.0 meters head, and Well C = 8.5 meters head in the given figure below. The distance between well A and well B is 275 meters and the distance between well A and well C is 325 meters. Divide the line between the two wells with the highest and lowest head into equal parts of 0.1 meters head, then determine what distance each increment of 0.1 meters head represents.

![Diagram](image)
9. APPLICATIONS FOR SKI AREA OPERATIONS

In problems 189 – 194, answer the following questions.

189. If one front-loader operator can dig 4 cubic yards of dirt per minute, how many hours would it take three front-loader operators working together at the same rate to dig a snow making trench that requires removal of 5000 cubic yards of dirt?

190. If one front-loader operator can dig 6 cubic yards of dirt per minute, how many hours would it take two front-loader operators working together at the same rate to dig a snow making trench that requires removal of 3250 cubic yards of dirt?

191. It costs a ski resort $15/hour to hire one front-loader operator. If the ski resort plans to hire four front-loader operators each digging at a rate of 1 cubic yard of dirt per 20 seconds to dig 8920 cubic yards of dirt, how much in labor will it cost the ski resort?

192. It costs a ski resort $17.25/hour to hire one front-loader operator. If the ski resort plans to hire five front-loader operators each digging at a rate of 7 cubic yard of dirt per minute to dig 6890 cubic yards of dirt, how much in labor will it cost the ski resort?

193. A ski resort has budgeted $1525 for labor costs in digging a snowmaking trench. If the ski resort plans to hire one front-loader operator digging at a rate of 4 cubic yards per minute to dig a trench that requires the removal of 15,524 cubic yards of dirt, what is the maximum hourly rate for the front-loader operator the ski resort can afford to stay within budget?

194. A ski resort has budgeted $2400 for labor costs in digging a snowmaking trench. If the ski resort plans to hire two front-loader operators, each digging at a rate of 4 cubic yards per minute to dig a trench that requires the removal of 18,892 cubic yards of dirt, what is the maximum hourly rate for each front-loader operator the ski resort can afford to stay within budget?

Solutions to Module I:

1. 87  2. -15  3. 12  4. -32  5. -33  6. 15  7. -16  8. 45  9. 0  10. -434  11. 32  12. -12  13. -6  14. 31  15. Undefined  16. 0  17. 1  18. -1  19. 1  20. 16  21. \(-5^4 = -5 \cdot 5 \cdot 5 = -625\) and \((-5)^4 = (-5)\times (-5)\times (-5)\times (-5) = 625\). Since the exponent is even, parentheses change the sign. 22. \(-5^3 = -5 \cdot 5 \cdot 5 = -125\) and \((-5)^3 = (-5)\times (-5)\times (-5) = -125\). Since the exponent is odd, parentheses don’t change the sign. 23. 2  24. 9  25. Not a real number  26. -8  27. -7  28. 0  29. 0  30. 5  31. -10  32. -1320  33. 12  34. -3  35. 495  36. -77  37. 122  38. Undefined  39. 36  40. -31  41. 6  42. -27  43. 1  44. 300,000,000; Three hundred million  45. 5,760,000,000,000; Five trillion, seven hundred sixty billion  46. 2,140,000; Two million, one hundred forty thousand  47. 13,040,000,000,000; Thirteen trillion, forty billion  48. 92,920,000; Ninety-two million, nine hundred twenty thousand  49. 0.00000000013; Three hundred-billionths  50. 0.0000000075; Seventy-five billionths  51. 0.00000000003; Three ten-billionths  52. \(2.3 \times 10^6\)  53. \(3.54 \times 10^8\)  54. \(2.47 \times 10^{13}\)  55. \(5.3 \times 10^9\)  56. \(1.023 \times 10^8\)  57. \(1.67 \times 10^{-27}\)  58. \(6.5 \times 10^{-9}\)  59. \(2.5 \times 10^{12}\)  bytes  60. \(2 \times 10^{14}\) gallons
61. 32 grains 62. 150 ounces vi 63. 12 drams 64. 240 grains 65. 16 drams 66.  24 fluid drams 67. 15 minims
68. 0155 hours 69. 1140 hours 70. 1932 hours 71. 8:12 AM 72. 11:25 PM 73. 12:34 PM
74. 36,792,000; 2,722,608,000; 2,943,360,000 75. 15 minims
76. 0155 hours 77. 1140 hours 78. 12:34 PM
79. 40.5 rads 80. 3221 grams 81. 4.79 or approximately 5 tablets
82. MFWS is 1.5 miles/hour 83. MFWS is 14 miles/hour 84. MFWS is 9 miles/hour
85. 2.57 pounds upward 86. Resultant force is 0 pounds, meaning that the weight is suspended
87. $482 88. $12,833 89. -$1197 90. -$46.83 ($46.83 overdrawn)
91. $18 92. $706 93. $176,180 94. $2854 95. $2400 96. $42,100
97. $327 reimbursement 98. $264.38 reimbursement 99. -$33 100. -$248 (a debt of $248)
101. a)  -$119  b)  -$139  102. $67 103. $2,222,570 104. $7767 105. $7767
106. $13,268; profit 107. -$55,551 108. $7.00 109. $8.00 110. $10.00
111. $11.00 112. $12.00 113. $14 114. $25 115. $4 116. $28 117. a) $804.81 b) $2,261.79 c) $443.58 d)
$3,510.18 118. a) 85 minutes   (1 hour 25 min.)  b)  $11.00  c)  $13.75  d)  $0.229  e)  $19.48  f)
$14.00  g)  $33.48  h)  $0.837  i)  $0.805  j) Buy the Cheesecake  119. $68.88; $1.78; $0.98; 5.625 lb; $2.76; 13.563 lb;
$66.13; $4.88; 1.358137684; $1.83; $4.75; $1.78
120. Forty-eight million, nine hundred twenty-one thousand, one hundred thirty-eight
121. Ten trillion, two hundred eighty-four billion, four hundred twenty-million, two hundred thirty-nine thousand, one hundred thirty-two
122. One quadrillion, two hundred thirty-eight trillion
123. Sixty-seven quintillion
124. 885 is divisible by 3 and 5 125. 10,800 is divisible by 2, 3, 4, 5, 6, 9, and 10
134. Factors: 1, 2, 17, 34 135. Factors: 1, 3, 9 136. Factors: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
137. Factors: 1, 2, 47, 94 138. Factors: 1, 3, 23, 69 139. Factors: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32,
48, 96 140. Factors: 1, 23 141. Factors: 1, 2, 41, 82 142. 112 143. 181 144. 603 145. 388
151. Mean: 25.455, Median: 17, Mode: 14 152. 67 sheets 153. 125 pieces of paper
154. 300 pieces of paper 155. 3.75” by 7.25” 156. 9.25” by 9.25” 157. 8.75” by 11.25”
158. $7.81/watt 159. $8.38/watt 160. 1.83 hours or 1 hour and 50 minutes 161. 7 x 10^9
162. $1,336,156,631 per day 163. 4.6875 meters per 0.1 head 167. 18.97 meters per 0.1 head 168. 30.56 meters per 0.1 head
169. 6.94 hours 170. 4.51 hours 171. $7.81/watt 172. $8.38/watt 173. 0.001286 174. 0.0000003766 175. 0.0002928
176. 3,600,000 177. 8,760,000,000,000 178. 1.0 x 10^15
Module II
Ratio, Rate, Proportion & Percent

Ratios, rates, proportions and percents are used heavily in real-world applications. Emergency Medical Technicians and nurses use them to express the relationship between two substances and to help calculate medication dosages; lenders use them to help determine qualification status for a loan; accountants use them in bookkeeping; firefighters use them to determine wind speed at eye level and flow rate of a hose; professional chefs use them to calculate ingredients and determine quantities of menu items; early childhood educators use them to help children make comparisons between numbers of objects and to introduce probability; graphic designers use them to describe the resolution of output devices such as printers; photographers use them to help calculate f-stop (full stop) differences; solar photovoltaic (PV) installers use them to help calculate the cost for installing a solar PV system; Oil industry technicians use them when programming instruments and in designing tanks; and the ski industry uses them to determine the best way to plan ski resort facilities that will maximize comfort for patrons while minimizing their costs.

I. Ratio

A ratio in mathematics can be described as a comparison between two quantities using a quotient. “The ratio of \( a \) to \( b \)” can be expressed many different ways:
1. With a colon: \( a : b \)
2. As a fraction: \( \frac{a}{b} \)
3. With the division symbol: \( a \div b \)
4. With the word “to”: \( a \) to \( b \)

A ratio is said to be in simplest form if \( a \) and \( b \) are both integers AND \( a \) and \( b \) do not have a common factor other than 1.

Example 1: Write the following ratios in simplest form.

a) \( 10:26 \)
    \[
    2\frac{1}{2} \\
    \]

b) \( \frac{3}{2} \)

\[
3\frac{2}{5} \]

c) 0.5 to 0.75
Solutions to Example 1:

a) Since 10 and 26 have a common factor other than 1, we will divide both numbers by the greatest common factor 2, to get 5:13. Answer: 5:13

b) Here we just need to divide the mixed numbers to get our ratio in simplest form:

\[
\frac{2 \frac{1}{3}}{3 \frac{2}{5}} = \left( \frac{\frac{1}{3}}{\frac{2}{5}} \right) = \frac{7}{3} \div \frac{17}{5} = \frac{7}{3} \cdot \frac{5}{17} = \frac{35}{51} \, \text{Answer: } \frac{35}{51}
\]

c) Since \(a\) and \(b\) need to be integers, we need to multiply by an appropriate power of ten to eliminate all of the decimal numbers. In this case, since we need to move the decimal two places to the right in 0.75 to get a whole number, we will multiply both 0.5 and 0.75 by 100. We now get 50 to 75. Now we will simplify this result by dividing 50 and 75 by 25 to get 2 to 3. Answer: 2 to 3

A unit ratio is a ratio in which the denominator is 1. In order to write a ratio as a unit ratio, simply divide both the numerator and the denominator by the denominator.

Example 2: Write the following as unit ratios.

a) 26:10

\[
\frac{2 \frac{1}{3}}{3 \frac{2}{5}} = \left( \frac{\frac{1}{3}}{\frac{2}{5}} \right) = \frac{7}{3} \div \frac{17}{5} = \frac{7}{3} \cdot \frac{5}{17} = \frac{35}{51} \, \text{Answer: } \frac{35}{51}
\]

b) 0.5 to 0.75

\[
\frac{0.5}{0.75} = \left( \frac{\frac{1}{2}}{\frac{3}{4}} \right) = \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9} \, \text{Answer: } \frac{8}{9}
\]

c) 0.5 to 0.75 = (0.5/0.75) to (0.75/0.75) = Answer: 2 to 1

Another type of ratio is a probability. When outcomes are equally likely to occur, such as flipping a heads or tails on a fair coin, the theoretical probability or classical probability is the ratio of the number of favorable outcomes (what you would like to see happen) to the number of possible outcomes (what possibly could happen). For example the probability of flipping heads on a fair coin is \(\frac{1}{2}\) since there is only one way to flip a heads (one heads on a two-sided coin) and there are two possible outcomes (heads or tails).
II. Rate

A **rate** is a ratio that compares two numbers with differing units. Examples of rates would include miles per hour, gallons per minute, customers per hour, etc. When writing a rate it is important to include the units (miles, hours, gallons, minutes, customers, etc.). A **unit rate** is a rate in which the denominator is equal to 1 unit. It is helpful to compare unit rates when deciding which purchase is a “better buy”.

**Example 3:** Which of the following is a better buy? 2-twelve ounce soda’s for $1.59 or a 32 ounce soda for $2.29.

Solution to Example 3:
In this example we would like to compare the unit rates for each purchase in price per ounce. For 2-twelve ounce soda’s we are paying $1.59 for 24 ounces. The unit price then would be $1.59/24 ounces ≈ $0.066 per ounce. The unit price for the 32 ounce soda would be $2.29/32 ounces ≈ $0.072 per ounce. Comparing the two rates we find that the better buy is 2-twelve ounce sodas for $1.59. **Answer:** 2-twelve ounce sodas for $1.59 is the better buy

III. Proportion

Khan Academy Resources: [https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-proportional-rel/v/introduction-to-proportional-relationships](https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-proportional-rel/v/introduction-to-proportional-relationships)

A **proportion** is the equality of two or more ratios. “a is to b as c is to d” can be expressed as:

1. \(a:b::c:d\)
2. \(a:b = c:d\)
3. \(\frac{a}{b} = \frac{c}{d}\)

The proportion \(\frac{a}{b} = \frac{c}{d}\) is true if and only if \(a \cdot d = b \cdot c\), that is, the **cross products** are equal.

For example, the proportion \(\frac{2}{3} = \frac{8}{12}\) is true since \(2 \cdot 12 = 3 \cdot 8 = 24\), but the proportion \(\frac{3}{8} = \frac{2}{12}\) is not true since \(3 \cdot 12 \neq 8 \cdot 2\) or \(36 \neq 16\). When two pairs of numbers, such as 2, 3 and 8, 12 have the same ratio, we say that they are **proportional**.

**Example 4:** Determine whether the two pairs of numbers are proportional.

a) 4, 5 and 6, 7
b) 12, 7 and 84, 49
Solution to Example 4:

a) We want to know if \( \frac{4}{5} = \frac{6}{7} \). Since \( 4 \cdot 7 \neq 5 \cdot 6 \) (28 \neq 30), this is not a true proportion (i.e., not proportional).

b) We want to know if \( \frac{12}{7} = \frac{84}{49} \). Since \( 12 \cdot 49 = 7 \cdot 84 = 588 \), this is a true proportion (i.e., proportional).


To solve a proportion in which there is an unknown amount, call it “\( N \)”, typically we will use a process called cross-multiplication, that is, we take the cross products;

\[
\frac{a}{b} = \frac{c}{d} \quad \text{means} \quad a \cdot d = b \cdot c.
\]

It is important that when we solve applications involving rates that we set up our proportion so that the units are kept in line, otherwise we will end up with an incorrect answer. Also, there are many different ways to set up proportions that will lead to correct answers, as long as the units are kept in line.

**Example 5:** Solve the following proportion for \( N \):

\[
\frac{327}{8} = \frac{N}{32}
\]

Solution to Example 5:

\[
\frac{327}{8} = \frac{N}{32}
\]

We cross multiply to get \( 327 \cdot 32 = 8N \). Divide both sides by 7 to get \( \frac{327 \cdot 32}{8} = \frac{8N}{8} \). After reducing, we get \( N \approx 36.6 \). **Answer:** \( N \approx 36.6 \)

**Example 6:** A doctor orders Amoxicillin 350 mg. The drug available is Amoxicillin 250 mg/5 mL suspension. How many mL should be administered?

Solution to Example 6: This problem is asking us to find the amount of mL for 350 mg of Amoxicillin. We are given a rate of 250 mg/5mL. With the given rate we can write our proportion in the following manner:

\[
\frac{250 \text{mg}}{5 \text{mL}} = \frac{350 \text{mg}}{x \text{mL}}
\]

Cross-multiply to get \( 250x = 1750 \), then divide both sides of the equation by 250 to get \( x = 7 \). **Answer:** 7 mL should be administered
IV. Percent

Khan Academy Resources: [https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-intro_percents/v/describing-the-meaning-of-percent](https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-intro_percents/v/describing-the-meaning-of-percent)

Looking at the word “percent”, “Per” means “out of” and “cent” comes from the Latin word centum meaning 100, therefore, percent literally means “out of 100”, or a ratio out of 100.

Khan Academy Resources: [https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-percent_decimal_conversions/v/converting_decimals_to_percents-ex-1](https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-percent_decimal_conversions/v/converting_decimals_to_percents-ex-1)

Converting Percents To Decimals:
- Write the numerical value as a decimal number.
- Move the decimal two places to the left and remove the percent sign.

Example 7: Convert the following to decimal numbers:
- a) 24%
- b) 125%
- c) $\frac{3}{4}\%$

Solutions to Example 7:
- a) Moving the decimal to the left two places gives us 0.24. Answer: 0.24
- b) Moving the decimal to the left two places gives us 1.25. Answer: 1.25
- c) First writing the mixed number as a decimal, we get 4.75%. Next, Moving the decimal to the left two places gives us 0.0475. Answer: 0.0475

Converting Decimals to Percents:
- Move the decimal place two places to the right.
- Add the % sign.

Example 8: Convert the following to percents:
- a) 0.87
- b) 0.00058

Solutions to Example 8:
- a) Move the decimal point in 0.87 two places to the right to get 87%. Answer: 87%
- b) Move the decimal point in 0.00058 two places to the right to get 0.058%. Answer: 0.058%

Converting Percents to Fractions:
- Since percent means “out of 100” we can write: $n\% = \frac{n}{100}$.
- Make sure to reduce your fraction after you have converted from a percent.
Example 9: Convert the following percents to fractions and reduce, if necessary:
   a) 52%    b) 72.5%    c) $\frac{125}{3}$ %

Solutions to Example 9:
   a) $\frac{52}{100} = \frac{13}{25}$. \textbf{Answer: $\frac{13}{25}$}
   b) $\frac{72.5}{100} = \frac{725}{1000} = \frac{29}{40}$. \textbf{Answer: $\frac{29}{40}$}
   c) $\frac{125}{3} = \frac{125}{300} = \frac{5}{12}$. \textbf{Answer: $\frac{5}{12}$}

Converting Fractions to Percents:
   a. Convert the fraction to a decimal number, then the decimal number to a percent, OR,
   b. Simply multiply the fraction by 100%.

Example 10: Convert the following to percents:
   a) $\frac{8}{25}$    b) $1\frac{1}{3}$

Solutions to Example 10:
   a) $\frac{8}{25} \cdot 100\% = 8 \cdot 4\% = 32\%$. \textbf{Answer: 32\%}
   b) $1\frac{1}{3} = 1.333\ldots = 133.333\ldots\% = 13\frac{1}{3}\%$. \textbf{Answer: 13\frac{1}{3}\%}

V. Percent Applications

Khan Academy Resources: https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-percent-word-problems/v/solving-percent-problems

Most percent problems can be written in the form of a basic percent sentence in which one of the three following pieces of information is not given:

1) The percent (or rate),
2) The base amount
3) The part.
Example 11: In the sentence “47.5% of Americans will vote for Hillary Clinton in the 2016 Presidential Election.”
   a) Identify the rate.
   b) Identify the base amount.
   c) Identify the part.

Solutions to Example 11:
   a) Answer: 47.5% is the rate
   b) Answer: All Americans is the base amount
   c) Answer: Americans that will vote for Hillary Clinton in the 2016 Presidential Election is the part

Basic Percent Sentence:

Most percent problems can be translated to the following basic percent sentence:

   A “percent” of “a base amount” is “part of the base amount”.

Once we identify two of the three pieces of information in the basic percent sentence then we may use one of two methods to solve for the missing amount.

Solving Percent Problems using Direct Translation:
   a. Write the basic percent sentence: A “percent” of “a base amount” is “part of the base amount”.
   b. Translate the basic percent sentence to an algebraic sentence by replacing “of” with multiplication and “is” with an equal sign. Therefore your percent sentence looks like: 
   \[ R \text{ (for rate, in decimal form)} \times B \text{ (for base amount)} = P \text{ (for part)} \]
   c. Solve for the missing amount.
   d. Check to see if our answer makes sense.

Example 12: Use direct translation to answer the following. 52% of 150 is what number?

Solution to Example 12:
52% of 150 is what number translates to the equation: 0.52 \times 150 = number. The number is 78.
Answer: The number is 78

Solving Percent Problems using Proportion:
   a. If we solve the direct translation equation \[ R \text{ (for rate)} \times B \text{ (for base amount)} = P \text{ (for part)} \] we obtain a proportion: \[ R = \frac{P}{B} \]
b. Identify R, P, and B. Substitute the known values into the proportion and solve for the missing value.

c. Check to see if our answer makes sense.

Note: To remember where P and B go, think: “is” over “of”.

**Example 13:** Use the proportion method to answer the following. 70% of what number is 39.2?

Solution to Example 13:
Identify R, P and B. R = 70%, P = 39.2, B is unknown. Write a proportion: \( \frac{70}{100} = \frac{39.2}{B} \). Solve for B:

\[ 70B = 3920 \]

\[ B = 56 \]

**Answer:** The number is 56

**NOW YOU TRY:** What percent of 125 is 200?

**Answer:** 160%

**Percent word problems, Percent Increase, Percent Decrease:**

When solving percent word problems, it is helpful to identify the three key parts of the basic percent sentence (rate, base, part) then decide which method (direct translation or proportion) we would like to use. Sometimes percent problems can be misleading, for example, in the problem:

Jenny received a 6% raise this year. Her total annual salary now is $55,756. What was her former annual salary?

It is very easy to want to use only the numbers that are given in the problem to answer the question; however, this problem requires that we consider that Jenny’s former salary represents 100%, while the 6% raise is an increase on top of the 100% making the percent that we will use to answer the question 106%.

In addition to adding to 100%, it is often helpful to subtract from 100%. For example, if you purchase a bicycle at a 30% discount, you can interpret this as paying 70% (100% - 30%) of the original cost.

**Example 14: (Subtracting Percents from 100%)** A jacket with an initial price of $120 is marked 40% off. What is the discount price?

Solution to Example 14:
A 40% discount translates to paying 60% of the original price. Our percent sentence now becomes: 60% of $120 is the discount price. Using direct translation the basic percent sentence translates to \(0.6 \cdot 120 = \text{the discount price}\). By multiplying, we get the discount price = $72. Answer: The discount price is $72

**Example 15:** (Adding Percents to 100%) Typically at a restaurant a 15% tip is customary. If you have only $65 in your pocket to spend on a meal, how much should the meal cost before tip?

Solution to Example 15:
A 15% tip is an amount of increase. Thinking that the meal alone is 100%, with an additional 15% tip, the amount that we pay is 115% of the cost of the meal alone. Our percent sentence now becomes: 115% of the meal before tip is $65. Using direct translation and \(x\) to equal the meal, the basic percent sentence translates to \(1.15 \cdot x = 65\). By dividing both sides by 1.15, we get \(x = 56.52\). Answer: The cost of the meal before tip is $56.52

**Example 16:** (Sales Tax) In Eagle County, Colorado the sales tax rate is 8.9%. Suppose you purchase a TV at Costco that retails for $1099.99. How much of the total cost of the TV is sales tax?

Solution to Example 16:
In this case we do not need to add or subtract the given percent to or from 100%. This problem directly translates to: 8.9% of 1099.99 is the amount of sales tax. Using direct translation and \(x\) to equal the sales tax, the basic percent sentence translates to \(0.089 \cdot 1099.99 = x\). Multiplying and rounding our answer to the nearest penny we get \(x = 97.90\). Answer: The sales tax is $97.90

**Example 17:** (Commission) As a realtor you are earning a 2.5% commission rate for each sale. If you sold a home and earned $10,700 in commission, how much did the home sell for?

Solution to Example 17:
Again, we do not need to add or subtract the given percent to or from 100%. This problem directly translates to: 2.5% of cost of the home is $10,700. Using direct translation and \(x\) to equal the cost of the home, the basic percent sentence translates to \(0.025 \cdot x = 10700\). Dividing both sides by 0.025 we get \(x = 428,000\). Answer: The home sold for $428,000

When solving *percent increase* or *percent decrease* (discount) problems we can refer to the basic percent sentence and then rephrase it:
PERCENT INCREASE: A “percent of increase” of “an initial amount” is “the amount of increase”.

PERCENT DECREASE: A “percent of decrease (or discount)” of “an initial amount” is “the amount of decrease (or discount)”.

With the sentence above we can solve any percent increase or decrease problems by method of direct translation, or proportion.

Example 18: Suppose gas prices in Colorado increase from $2.29 per gallon to $4.15 per gallon. What is the percent of increase in gas prices?

Solution to Example 18:  
We need to know the initial amount ($2.29), and the amount of increase ($4.15 – $2.29 = $1.86). Now we can relate these values to the basic percent sentence: The percent increase of $2.29 is $1.86. Using direct translation and \( x \) to equal the percent increase, the basic percent sentence translates to \( x \cdot 2.29 = 1.86 \). Dividing both sides by 2.29 and rounding to the nearest thousandth we get \( x = 0.812 \). Since this answer is a percent given in decimal form, we will convert our answer to percent form: \( x = 81.2\% \) Answer: The percent increase in gas prices is 81.2%

Homework Set:

In problems 1 – 12, express the ratios in lowest terms. Remember, a ratio must have the same units, so if units are involved, then you must convert them to the same unit.

1. 4:16  
2. 6 to 9  
3. \( \frac{32}{20} \)  
4. 15:5  
5. 2 ½ to 10  
6. \( \frac{5}{2.5} \)  
7. 0.25:0.635  
8. 3 inches to 1 foot  
9. 45 minutes to 3 hours  
10. $5.00 to 50¢  
11. The dimensions of a picture are 5” width by 7” height. A photographer wants to enlarge it to an 8” width by 10” height. What is the ratio of the width of the picture to the width of the enlargement?
12. The dimensions of a picture are 8” width by 10” height. A photographer wants to enlarge it
to an 20” width by 24” height. What is the ratio of the height of the picture to the height
of the enlargement?
In problems 13 – 16, solve the proportion for x. Round decimal answers to the nearest
hundredth.

x 7
=
5 10
3.4 x
=
14.
8
7
6.3 8
=
15.
x
5
3
3
5
16. 4 =
7
x
13.

In problems 17 – 20, write the percent as a decimal number.
17. 45%
18. 132%
19. 0.5%
20.

3
%
8

In problems 21 – 24, write the percent as a fraction in simplest form.
21. 30%

2
3

22. 16 %
23. 12.5%
24. 475%
In problems 25 – 28, write the decimal number as a percent.
25.
26.
27.
28.

0.65125
0.0037
0.2
1.0275

In problems 29 – 32, write each fraction as a percent.
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29. \( \frac{7}{50} \)
30. \( \frac{15}{8} \)
31. \( \frac{13}{20} \)
32. \( \frac{2}{3} \)

In problems 33 – 38, answer the basic percent sentence problems. Round decimal answers to the nearest hundredth.

33. 34% of 852 is what number?
34. 98% of 38 is what number?
35. 82% of what number is 24?
36. 55.5% of what number is 5025?
37. 554 out of 600 is what percent?
38. What percent of 95 is 300?

In problems 39 – 48, answer the following percent applications. Round decimal answers to the nearest hundredth.

39. Jenny received a 6% raise this year. Her total annual salary now is $55,756. What was her former annual salary?
40. Josh paid $34,937.50 for a new car, including sales tax. If the sales tax rate is 7.5%, what was the price of the car?
41. A company finds that they are spending 4 ½% of their monthly revenue on office supplies. If the company averages $28,500 in monthly revenue, how much is spent on office supplies?
42. A company finds that they spent $3,350 on vehicle maintenance. If the company averages $22,324 in monthly revenue, what percent of the monthly revenue is spent on vehicle maintenance?
43. A company’s costs in June are $4320. Their costs in July are $6100. What is the percent increase in costs from the months of June to July?
44. At the beginning of last year CMC’s math department budget was 15% of the total budget. This year it was 10% of the total budget. What percent did the CMC’s math department budget decrease from last year?
45. By discontinuing their home phone service and only using cellular phones, the Vargas family was able to cut their costs from $162 to $73 per month. What is the percent decrease in phone costs?
46. The Vargas and Gutierrez partnership allocates profits (or losses) 60% to Vargas and 40% to Gutierrez. Osborn enters the partnership and receives a 20% share of profits (or losses). What share of profits (or losses) does Vargas now share?
47. The Vargas and Gutierrez partnership allocates profits (or losses) 55% to Vargas and 45% to Gutierrez. Osborn enters the partnership and receives a 30% share of profits (or losses). What share of profits (or losses) does Gutierrez now share?
48. Bronn Trucking purchased mechanical parts costing a total of $32,000. The vendor’s terms offered a 5% discount on any part of the total cost paid within five business days. Bronn Trucking paid $20,500 (amount after discount) on the third business day.
   a. How much discount did Bronn Trucking receive?
   b. How much did Bronn Trucking owe after making the payment?

1. APPLICATIONS FOR EMT/NURSING/MEDICAL ASSISTANT

FUN FACT: In the medical field, ratios are used to express the relationship between two substances in a solution. Epinephrine with a local anesthetic, for example, is used for subcutaneous injection (an injection from a needle placed just under the skin). It is supplied in a ratio of 1:100,000 or 1 part epinephrine to 100,000 parts solution. Rates, or specifically, unit rates (rates given per 1 unit) are used to describe quantities with different units such as milligrams per tablet when calculating medications. In problems 49 – 52 we will get practice in finding unit rates.

In problems 49 – 52, write each rate as a unit rate.

49. \( \frac{2600 \text{ mg}}{8 \text{ tablets}} \)

50. \( \frac{250 \text{ mg}}{0.5 \text{ tablets}} \)

51. \( \frac{5 \text{ g}}{100 \text{ mL}} \)

52. \( \frac{60 \text{ mEq}}{15 \text{ mL}} \)

FUN FACT: In the medical field, percents denote the strength of the medication in an ointment, solution, or intravenous fluid. For example, a 3% solution describes 3 parts of a particular substance, or solute, to 100 parts of another substance, called the diluent. We would say that the percent strength of the solution is 3%. The higher the percentage on the drug label, the higher the strength of the solution or drug. In problems 53 – 56, we will practice recognizing higher-strength dosages. In problems 57 – 61, we will calculate the amount of solute given the percent strength and the diluent.

In problems 53 – 56, pairs of strength percentages for a particular drug are given, identify the higher-strength doses.

53. 0.1% or 0.01%
54. 2.5% or 0.25%
55. 0.25% or 0.3%
56. 0.01% or 0.03%

In problems 57 – 61, determine how many grams of drug are found in each solution. For example, dextrose 5% and water ($D_{2}W$ or 5% $D/W$) has 5 grams of dextrose per 100 mL.

57. How many grams of dextrose are found in 1,000 mL of a 5% solution?
58. How many grams of sodium are found in 1,000 mL of a 3% solution?
59. How many grams of sodium are found in 1,000 mL of a 0.9% solution?
60. How many grams of dextrose are found in 500 mL of a 2.5% solution?
61. How many grams of dextrose are found in 250 mL of a 5% solution?

In problems 62 – 74, solve by setting up a proportion. Show your work!

62. A patient is to receive 15 mg of a drug. The tablets available are 10 mg tablets. How many tablets should be administered?
63. A patient is to receive 0.5 gm (500 mg) of a drug. The drug is available in a 250 mg/5 mL solution. How many mL of medication should be administered?
64. The physician orders Furosemide 80 mg. The drug available is Furosemide 40 mg/5mL solution. How many mL should be administered?
65. The physician orders Meperidine 35 mg. The drug available is Meperedine 100 mg/1mL solution. How many mL should be administered?
66. The physician orders Aspirin 325 mg. The drug available is Aspirin 81 mg/tablet. How many tablets should be administered?
67. The physician orders KCL 20 mEq. The drug available is KCL 60 mEq/15mL solution. How many mL should be administered?
68. The physician orders Amoxicillin 300 mg. The drug available is Amoxicillin 250 mg/5mL suspension. How many mL should be administered?
69. The physician orders Penicillin 250,000 units. The drug available is Penicillin 1,000,000 units/5 mL solution. How many mL should be administered?
70. The physician orders Furosemide 80 mg. The drug available is Furosemide 40 mg/5mL solution. How many mL should be administered?
71. The physician orders Dexamethasone 0.75 mg. The drug available is Dexamethasone 0.25 mg/tablet. How many tablets should be administered?
72. The physician orders Digoxin 0.5 mg. The drug available is Digoxin 0.125 mg/tablet. How many tablets should be administered?
73. You have a vial with 30 mL of a liquid solution. If the average dose is 50 mL, how many doses are available?
74. A doctor prescribes 3 pills per day. How many pills will the patient need for 21 days?

2. APPLICATIONS FOR FIRE SCIENCE

FUN FACT: In fire science, flow rates describe the speed at which water is flowing. They are described as a unit rate in gallons per minute (gpm).

In problems 75 – 77, calculate the flow rate in gallons per minute. Round your answer to the nearest hundredth of a gallon per minute.

75. It takes 2 minutes to fill a 5 gallon bucket. What is the flow rate?
76. It takes 3 minutes and 40 seconds to fill a 100-gallon bucket. What was the flow rate?
77. It takes 8 ½ minutes to fill a 1 gallon container. What is the flow rate?

In problems 78 – 79, answer the ratio questions. Write your ratio in simplest form.

78. The Mara Bella District has 6 engines. The Baldy District has 5 engines. What is the ratio of engines in the Mara Bella to the Baldy districts?
79. In the U.S. in 2008, there were 34 wildfires in the month of July and 12 wildfires in the month of November. What is the ratio of wildfires in July to wildfires in November?

In problems 80 – 81, answer the following question by setting up and solving a proportion. Round your answer to the nearest hundredth of a gallon. Hint: 35:1 gas-to-oil mixture can mean 35 parts gas and 1 part oil OR 35 parts gas and 36 total parts OR 1 part oil and 36 total parts.

80. Sabrina has a 2-1/2 gallon container. She is going to fill it with a 40:1 gas-to-oil mixture. How much gas does she need to put into the container?
81. Sabrina has a 2-1/2 gallon container. She is going to fill it with a 40:1 gas-to-oil mixture. How much oil does she need to put into the container?

FUN FACT: A commonly used ratio in firefighting for relating 20-foot, eye-level, and litter-level (ground-level) wind speeds is 4:3:1. That is, the eye-level winds are 3/4 the 20-foot wind speed, and the litter-level winds are 1/3 the eye-level wind speed.

In problems 82 – 83, use this 4:3:1 ratio (20-ft wind: eye-level wind: litter-level wind) to answer the following wind speed questions.

82. You receive a RAWS (Remote Automatic Weather Stations) report of 20-foot winds blowing at 24 mph. What are the winds at eye-level?
83. What are the litter-level winds for the same report in the previous problem?
FUN FACT: Percentages are useful for a number of fire science applications. One of these applications is estimating live fuel moisture. We will see how to calculate the moisture content using equations and percentages in a later Module.

In problems 84 – 86, answer the percent questions.

84. When the crew leaves for a fire, they have a full tank of foam agent. This tank holds a total of 4.80 gallons. When they return to the station, they find only 0.91 gallons. What percentage of the total foam is left? Round your answer to the nearest percent.
85. Using the information from problem 84, what percentage of the foam was used? Round your answer to the nearest percent.
86. An engine operator notices that 40% of the tank water has been used in 1 hour. At that rate, how long will it take to empty the tank?

FUN FACT: Probability is the likelihood that an event will happen and can be expressed in terms of percent. For example, in Fire Science, if there is a 40 percent chance of a spot fire starting, that means that out of 100 glowing embers that fly off, 40 embers will likely start spot fires, and 60 will not.

In problems 87 – 90, answer the following questions involving probability.

87. The probability of ignition is 80%. How many ignitions are likely to occur if 90 glowing firebrands land on receptive fuel?
88. The probability of ignition is 45%. How many ignitions are likely to occur if 80 glowing firebrands land on receptive fuel?
89. If 32 ignitions occurred out of 50 glowing firebrands landing on receptive fuel, what is the probability of ignition? Write your answer as a percent.
90. The probability of ignition is 95%. How many glowing firebrands landed on receptive fuel if there were 120 ignitions? Round your answer to the nearest ignition.

3. APPLICATIONS FOR ACCOUNTING

Note: Although accountants and bookkeepers may not be directly involved in the decision process for home loans, they do deal with ratios and rates on a daily basis. The next series of examples are meant as practice for learning and understanding ratios, in addition to becoming familiar with how lenders decide whether to process or deny a loan. In a later Module, we will be furthering our investigation with loans when we introduce equations that calculate loan payments.

In problems 91 – 93, calculate each ratio as a unit ratio and interpret it’s meaning. Round decimal answers to the nearest hundredth.

91. A family has a total debt of $8,952 and a gross income of $58,930. What is its debt-to-income ratio?
92. A family has a gross income of $47,325 and a total debt of $9780. What is its debt-to-income ratio?

93. A family is seeking a mortgage to purchase a new home. If it is approved for the mortgage, the new payment will be $1645 per month. If the gross monthly income is $4850, what is the payment-to-income ratio?

FUN FACT: There are several factors that help lenders determine whether or not to process or deny a loan application. The criteria most commonly considered in this decision are debt-to-income ratio (DTI) and credit score. The debt-to-income ratio helps banks measure how easy it will be for a loan applicant to repay the loan. The DTI is split up into two ratios called the front-end ratio and back-end ratio for underwriting standards. The front-end ratio is the ratio of the total monthly house payment, which includes principal, interest, taxes and insurance (A.K.A. PITI payment) to gross monthly income. The back-end ratio is the ratio of the total monthly debt payments to gross monthly income, that is, the back-end ratio includes all debt, for example, other installment loans such as car payments or student loans, finance obligations such as credit cards, alimony and child support, as well as property taxes and home owner’s insurance.

### Factors for Loan Qualification

<table>
<thead>
<tr>
<th></th>
<th>Conventional</th>
<th>VA (Veteran Affairs)</th>
<th>FHA (Federal Housing Authority)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front-end ratio should not exceed</td>
<td>0.28</td>
<td>N/A</td>
<td>0.29</td>
</tr>
<tr>
<td>Back-end ratio should not exceed</td>
<td>0.36</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>Credit score should be</td>
<td>650 or higher</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

In problems 94 – 102, Use the “Factors for Loan Qualification” table above to answer the following questions.

94. A loan applicant is trying to qualify for a conventional loan. The applicant’s gross monthly income is $6790. If the applicant gets the loan, the monthly PITI payment will be approximately $1850. The applicant’s monthly debt excluding the monthly PITI payment is $785. The applicant has a credit score of 700. Does the applicant qualify?

95. A loan applicant is trying to qualify for a conventional loan. The applicant’s gross monthly income is $4380. If the applicant gets the loan, the monthly PITI payment will be approximately $1435. The applicant’s monthly debt excluding the monthly PITI payment is $340. The applicant has a credit score of 680. Does the applicant qualify?

96. A loan applicant is trying to qualify for a VA loan. The gross monthly income is $3945. If the applicant gets the loan, the monthly PITI payment will be approximately $1290. The credit score is 660. The applicant has the following monthly debt payments: Credit card 1 = $25, Credit card 2 = $32, Car loan = $125, Student loan = $90
Does the applicant qualify?

97. A loan applicant is trying to qualify for a FHA loan. The gross monthly income is $3530. If the applicant gets the loan, the monthly PITI payment will be approximately $785. The credit score is 625. The applicant has the following monthly debt payments:
   Credit card = $40, Car loan = $249, Student loan = $65

Does the applicant qualify?

98. A loan applicant has a gross monthly income of $9800. What is the maximum monthly PITI payment that would meet the front-end ratio qualification for a conventional loan?

99. A loan applicant has a gross monthly income of $3250. What is the maximum monthly PITI payment that would meet the front-end ratio qualification for a FHA loan?

100. A loan applicant has a gross monthly income of $7550. What is the maximum debt that would meet the back-end ratio qualification for a conventional loan?

101. A loan applicant has a gross monthly income of $2100. What is the maximum debt that would meet the back-end ratio qualification for a VA loan?

102. A loan applicant has a gross monthly income of $4200. What is the maximum debt that would meet the back-end ratio qualification for a FHA loan?

4. APPLICATIONS FOR CULINARY ARTS

FUN FACT: In the food industry, managers use ratios and rates to figure costs and to help determine the number of waiters that should be scheduled for a given shift. A common unit rate used in a restaurant is the turnover rate, which is the ratio of the total number of customers to the number of seats available

In problems 103 – 106, find the turnover rate. Round decimal answers to the nearest hundredth.

103. If 250 customers are served during lunch in a restaurant with 100 seats, what is the turnover rate?

104. If 125 customers are served during dinner in a restaurant with 32 seats, what is the turnover rate?

105. If 65 customers are served during breakfast in a restaurant with 80 seats, what is the turnover rate?

106. If 390 customers are served during lunch in a restaurant with 152 seats, what is the turnover rate?

In problems 107 – 109, calculate the rate in cost per ounce. Round decimal answers to the nearest hundredth.

107. One liter (33.8 oz.) of vodka costs $12.95. What is the cost per ounce?

108. One gallon (128 oz.) of O.J. costs $8.50. What is the cost per ounce?
109. If milk is purchased at $13.65 for a 4 gallon (128 oz. = 1 gal.) case, what is the cost of per ounce? What is the cost for 12 ounces?

In problems 110 – 115, the following ingredients and quantities are for a Chicken Marsala recipe that yields 20 portions. Use proportions to convert the recipe for Chicken Marsala below to produce 70 portions.

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Amount for 20 portions</th>
<th>Amount for 70 portions (include units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>110. Butter</td>
<td>5 oz.</td>
<td></td>
</tr>
<tr>
<td>111. Chicken</td>
<td>100 oz.</td>
<td></td>
</tr>
<tr>
<td>112. Mushrooms</td>
<td>2 ¼ lb.</td>
<td></td>
</tr>
<tr>
<td>113. Flour</td>
<td>4 oz.</td>
<td></td>
</tr>
<tr>
<td>114. White stock</td>
<td>3 ½ c.</td>
<td></td>
</tr>
<tr>
<td>115. Marsala</td>
<td>400 mL</td>
<td></td>
</tr>
</tbody>
</table>

FUN FACT: The food cost percentage of the total dollar amount of food sales is the cost of food. Knowing and understanding food cost percentage will help a restaurant owner determine the pricing for each menu item. In general, the lower the food cost percentage, the higher the profit percentage. The menu mix (or sales mix) percentage of the total food items on the menu sold is the amount sold of a specific food item. Ideal (theoretical) food cost percentage of the total revenue is the total food cost. The percentage yield of the total amount needed per customer is the amount of the standard portion.

In problems 116 – 120, use the fun fact above and any method to answer the questions.

116. Given that one liter (33.8 oz.) of vodka costs $12.95 and one gallon (128 oz.) of O.J. cost $8.50, calculate the cost of each ingredient and the total cost of a beverage given the following information. Recipe calls for:
   2 oz. of vodka            Cost = _____
   4 oz of O.J.              Cost = _____
   Total Cost = _____

117. If the selling price for the beverage in problem (116) above is $3.95, calculate the food cost percent.

118. If an item has a food cost percent of 30%, and a Selling price of $9.99, what is the food cost of the item? Round your answer to the nearest penny.

119. If an item has a food cost of $4.00 and a desired food cost percent of 40%, what is the preliminary sales price?

120. Calculate the beverage cost percent given the following information:
   Beverage Cost $2,103   Beverage Sales $9,562
In problems 121 – 124, use the given information to calculate the Food Cost %, and Menu Mix % for each of the following items. Note: You will need to consider all of Food Items A – D.

<table>
<thead>
<tr>
<th>Food Item</th>
<th>Selling Food Cost</th>
<th>Selling Food Price</th>
<th># Sold</th>
<th>Cost %</th>
<th>Menu Mix %</th>
</tr>
</thead>
<tbody>
<tr>
<td>121. A</td>
<td>3.00</td>
<td>10.00</td>
<td>70</td>
<td>_____%</td>
<td>______%</td>
</tr>
<tr>
<td>122. B</td>
<td>4.00</td>
<td>12.00</td>
<td>30</td>
<td>_____%</td>
<td>______%</td>
</tr>
<tr>
<td>123. C</td>
<td>5.00</td>
<td>15.00</td>
<td>20</td>
<td>_____%</td>
<td>______%</td>
</tr>
<tr>
<td>124. D</td>
<td>6.00</td>
<td>17.00</td>
<td>80</td>
<td>_____%</td>
<td>______%</td>
</tr>
</tbody>
</table>

In problems 125 – 129, use the given information information to find the menu mix percentage. Note: You will need to consider all of the food items listed.

<table>
<thead>
<tr>
<th>Item</th>
<th># Sold</th>
<th>Menu Mix %</th>
</tr>
</thead>
<tbody>
<tr>
<td>125. Fish</td>
<td>522</td>
<td>_____%</td>
</tr>
<tr>
<td>126. Beef</td>
<td>670</td>
<td>_____%</td>
</tr>
<tr>
<td>127. Chicken</td>
<td>448</td>
<td>_____%</td>
</tr>
<tr>
<td>128. Veggie</td>
<td>198</td>
<td>_____%</td>
</tr>
<tr>
<td>129. Game</td>
<td>267</td>
<td>_____%</td>
</tr>
</tbody>
</table>

In problems 130 – 134, use the given information to calculate the Food Cost % for each item. Round each percent to the nearest tenth.

<table>
<thead>
<tr>
<th>Item</th>
<th>Food Cost</th>
<th>Selling Price</th>
<th>Food Cost %</th>
</tr>
</thead>
<tbody>
<tr>
<td>130. STEAK</td>
<td>$8.00</td>
<td>$22.00</td>
<td>_____%</td>
</tr>
<tr>
<td>131. LAMB LOIN</td>
<td>$10.00</td>
<td>$26.00</td>
<td>_____%</td>
</tr>
<tr>
<td>132. BAKED HAM</td>
<td>$4.00</td>
<td>$17.95</td>
<td>_____%</td>
</tr>
<tr>
<td>133. VEAL LOIN</td>
<td>$6.80</td>
<td>$22.00</td>
<td>_____%</td>
</tr>
<tr>
<td>134. CHICKEN</td>
<td>$3.20</td>
<td>$15.95</td>
<td>_____%</td>
</tr>
</tbody>
</table>

In problems 135 – 141, use the given information to calculate each of the missing values. Round your answer to the nearest hundredth of a percent.

<table>
<thead>
<tr>
<th>Item</th>
<th># Sold</th>
<th>Food Cost</th>
<th>Selling Price</th>
<th>Food Cost</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>135. STEAK</td>
<td>500</td>
<td>$8.00</td>
<td>$22.00</td>
<td>$________</td>
<td>$________</td>
</tr>
<tr>
<td>136. LAMB</td>
<td>100</td>
<td>$10.00</td>
<td>$26.00</td>
<td>$________</td>
<td>$________</td>
</tr>
<tr>
<td>137. HAM</td>
<td>300</td>
<td>$4.00</td>
<td>$17.95</td>
<td>$________</td>
<td>$________</td>
</tr>
<tr>
<td>138. VEAL</td>
<td>350</td>
<td>$6.80</td>
<td>$22.00</td>
<td>$________</td>
<td>$________</td>
</tr>
<tr>
<td>139.CHALTONC</td>
<td>750</td>
<td>$3.20</td>
<td>$15.95</td>
<td>$________</td>
<td>$________</td>
</tr>
<tr>
<td>140. TOTAL MENU</td>
<td></td>
<td>$________</td>
<td>$________</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

141. IDEAL (theoretical) Food Cost % ______
In problems 142 – 145, use the given information to calculate the number of pounds needed to serve a forecasted weekly customer count of 2,000 covers. Note: 16 oz. = 1 lb.

<table>
<thead>
<tr>
<th>ITEM</th>
<th>STANDARD</th>
<th>YIELD %</th>
<th>MENU MIX</th>
<th>Forecast # Sold</th>
<th>POUNDS NEEDED</th>
</tr>
</thead>
<tbody>
<tr>
<td>STEAK</td>
<td>12 oz.</td>
<td>75 %</td>
<td>32%</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>LAMB LOIN</td>
<td>8 oz.</td>
<td>50 %</td>
<td>22%</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>BAKED HAM</td>
<td>6 oz.</td>
<td>65 %</td>
<td>28 %</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>VEAL LOIN</td>
<td>6 oz.</td>
<td>85 %</td>
<td>18 %</td>
<td>______</td>
<td>______</td>
</tr>
</tbody>
</table>

In problems 146 – 147, use the given information to calculate the theoretical (total) food cost and the theoretical (total) food cost percentage.

<table>
<thead>
<tr>
<th>ITEM</th>
<th>NUMBER SOLD</th>
<th>SELLING PRICE</th>
<th>FOOD COST %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamburgers</td>
<td>2,000</td>
<td>$8.75</td>
<td>25 %</td>
</tr>
<tr>
<td>Fish</td>
<td>1,000</td>
<td>$14.00</td>
<td>32 %</td>
</tr>
<tr>
<td>Chicken</td>
<td>1,600</td>
<td>$11.50</td>
<td>20 %</td>
</tr>
</tbody>
</table>

146. Theoretical (Total) Food Cost $____________________

147. Theoretical (Total) Food Cost Percentage ____________%

In problem 148 below, use the given information to fill in the missing numbers.

<table>
<thead>
<tr>
<th>Day</th>
<th>Date</th>
<th>Daily Purchases</th>
<th>Daily Sales</th>
<th>Daily Cost %</th>
<th>To-Date Purchases</th>
<th>To-Date Sales</th>
<th>To-Date Cost %</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>8/1</td>
<td>$ 960.56</td>
<td>$2475</td>
<td>______</td>
<td>______</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>Th</td>
<td>8/2</td>
<td>$1806.85</td>
<td>$3015</td>
<td>______</td>
<td>______</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>Fri</td>
<td>8/3</td>
<td>$2005.15</td>
<td>$5314</td>
<td>______</td>
<td>______</td>
<td>______</td>
<td>______</td>
</tr>
<tr>
<td>Sat</td>
<td>8/4</td>
<td>$1358.55</td>
<td>$6077</td>
<td>______</td>
<td>______</td>
<td>______</td>
<td>______</td>
</tr>
</tbody>
</table>

In problems 149 – 151, use the given information to calculate the Menu Mix % for each of the following entrees. Based on the Menu Mix % and the # On-Hand, calculate the total “# Needed” for each item and the “# to Produce” based on a forecast of 400 diners.

<table>
<thead>
<tr>
<th>Item</th>
<th># Sold</th>
<th>Menu Mix %</th>
<th># Needed</th>
<th># On-Hand</th>
<th># To Produce</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beef</td>
<td>550</td>
<td>______%</td>
<td>______</td>
<td>13</td>
<td>______</td>
</tr>
<tr>
<td>Chicken</td>
<td>700</td>
<td>______%</td>
<td>______</td>
<td>28</td>
<td>______</td>
</tr>
<tr>
<td>Fish</td>
<td>350</td>
<td>______%</td>
<td>______</td>
<td>0</td>
<td>______</td>
</tr>
</tbody>
</table>
In problem 152, calculate the missing information on the Income Statement using the information below. Each percentage is calculated from sales.

152.

### INCOME STATEMENT
THE COST CONTROL RESTAURANT
YEAR ENDING DECEMBER 31, 2010

<table>
<thead>
<tr>
<th>Category</th>
<th>Amount</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SALES</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Food</td>
<td>$1,400,000</td>
<td>100%</td>
</tr>
<tr>
<td>b) Beverage</td>
<td>$450,000</td>
<td></td>
</tr>
<tr>
<td>c) Total Sales</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td><strong>COST OF SALES</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) Food</td>
<td>$450,000</td>
<td>31%</td>
</tr>
<tr>
<td>e) Beverage</td>
<td>$95,000</td>
<td>22%</td>
</tr>
<tr>
<td>f) Total Cost of Sales</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td><strong>GROSS PROFIT</strong></td>
<td></td>
<td></td>
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<tr>
<td>g)</td>
<td>$95,000</td>
<td>31%</td>
</tr>
<tr>
<td><strong>CONTROLLABLE EXPENSES</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h) Payroll</td>
<td>$460,000</td>
<td></td>
</tr>
<tr>
<td>i) Payroll Taxes and Employee Benefits</td>
<td>$95,000</td>
<td></td>
</tr>
<tr>
<td>j) Employee Meals</td>
<td>$2,000</td>
<td>2%</td>
</tr>
<tr>
<td>k) Direct Operating Expenses</td>
<td>$13,000</td>
<td>7%</td>
</tr>
<tr>
<td>l) Entertainment</td>
<td>$13,000</td>
<td>3%</td>
</tr>
<tr>
<td>m) Advertising and Promotion</td>
<td>$13,000</td>
<td>2%</td>
</tr>
<tr>
<td>n) Utilities</td>
<td>$33,000</td>
<td>5%</td>
</tr>
<tr>
<td>o) Administrative and General Fees</td>
<td>$13,000</td>
<td></td>
</tr>
<tr>
<td>p) Repairs and Maintenance</td>
<td>$21,000</td>
<td>2%</td>
</tr>
<tr>
<td>q) Total Controllable Expenses</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td><strong>PROFIT BEFORE RENT OR OCCUPANCY COSTS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r)</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td><strong>RENT OR OCCUPANCY COSTS</strong></td>
<td></td>
<td></td>
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<tr>
<td>s)</td>
<td>$300,000</td>
<td>31%</td>
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<tr>
<td><strong>PROFIT BEFORE DEPRECIATION</strong></td>
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<td>t)</td>
<td>100%</td>
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<tr>
<td><strong>DEPRECIATION</strong></td>
<td></td>
<td></td>
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<tr>
<td>u)</td>
<td>$13,000</td>
<td>4.2%</td>
</tr>
<tr>
<td><strong>NET PROFIT BEFORE INCOME TAX</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>v)</td>
<td>100%</td>
<td></td>
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</tbody>
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5. APPLICATIONS FOR EARLY CHILDHOOD EDUCATION

Early childhood educators help teach children how to compare and order numbers as well as understand the meaning of ratios, rates, proportions, percents, and probability. An early childhood educator must, at the very minimum, understand the following problems in order to create lessons that will effectively teach children about these concepts.

In problems 153 – 165, answer the following questions.

153. Convert the percent to a ratio: 58%
154. Convert the ratio to a percent: 46:100
155. Create a proportion from the set of numbers: 6, 1, 2, 3
156. Create a proportion from the set of numbers: 3, 91, 21, 13
157. Write the ratio in simplest form: 22:33
158. Write the ratio in simplest form: 104/24
159. Write the ratio in simplest form: 143 to 187
160. Write the percent of the shaded portion in the picture below:

![Shaded portion image]

161. Convert 55% to a decimal number.
162. Convert 0.38 to a percent.
163. Convert $\frac{3}{8}$ to a decimal number.
164. Convert 0.85 to a fraction in simplest form.
165. 30 is 60% of what number?
In problems 166 – 172, use the spinner above to answer the following questions. Write your probability as a fraction in reduced form. You may assume that each number is equally likely to be landed on.

166. What is the probability that the spinner will land on 5?
167. What is the probability that the spinner will land on an even number?
168. What is the probability that the spinner will land on a prime number?
169. What is the probability that the spinner will land on 2 or 3?
170. What is the probability that the spinner will land on a number less than or equal to 6?
171. What is the probability that the spinner will land on 1 and 3?
172. If you have a jar of jellybeans and you have the following colors: 9 red, 12 pink, 2 white, 14 black, and 3 orange, what is the probability that you will pick a red or white jelly bean?

6. APPLICATIONS FOR GRAPHIC DESIGN/PROFESSIONAL PHOTOGRAPHY

FUN FACT: A pixel, also known as picture element, is a single point or (x, y) coordinate in a raster image. A raster image, also known as a bitmap, is a way to display digital images, such as a computer screen. Pixels per inch, or PPI is a rate that is used to measure, for instance, the resolution on your computer monitor. It is important to note that there is no space between pixels. Dots per inch, or DPI is a rate that measures how many dots of ink or toner, and the blank spaces in between them a printer can place within an inch. The printed dots have space between them to make the color white, or no space between them to make the color black.

In problems 173 – 179, convert each to unit rates.

173. The IBM T220/T221 LCD monitors that were marketed from 2001–2005 reached 1020 pixels per 5 inches. What was this monitor’s PPI?
174. The Toshiba Portégé G900 Windows Mobile 6 Professional phone, launched in mid 2007,
came with a 3” WVGA LCD having “print-quality” pixel density of 2504 pixels every 8 inches. What was this phone’s PPI?
175. In June 2010, Apple Computer announced and launched the iPhone 4, with its “Retina” LED-backlit LCD boasting 163 pixels per half-inch. What was this phone’s PPI?
176. It has been observed that the unaided human eye can generally not differentiate detail beyond 100 pixels per 1/3 inch. What is the PPI?
177. A dot matrix printer, has a relatively low resolution, typically no higher than 22 ½ dots per quarter-inch. What is the DPI?
178. An inkjet printer sprays ink through tiny nozzles, and is typically capable of at least 3600 dots per foot. What is the DPI?
179. A laser printer applies toner through a controlled electrostatic charge, and may reach 1200 dots per 2/3 inches. What is the DPI?

FUN FACT: When a digital image is prepared for reproduction on a printing press, pixels are converted to dots. 300 pixels become 150 dots and spaces, as a result 300 PPI becomes roughly 150 DPI.

FUN FACT: Graphic designers use a proportion wheel to calculate proportional values of size, and percentage of enlargement or reduction.

In problems 180 – 183, solve the problem by setting up and solving a proportion. Show your work. Round decimal answers to the nearest hundredth.

180. A photograph that will be used in a newspaper column measures 2” width by 3.5” height. What height will the photograph be if the newspaper column is 5” wide?
181. A photograph that will be used in a newspaper column measures 5” width by 7” height. What width will the photograph be if the newspaper column is 4” high?

182. A photograph taken by a newspaper staff photographer measures 6” width by 8” height after being cropped. What will the width of the halftone that will appear in the daily newspaper if the height is three inches?

183. A 5” width by 6” height photograph is going to be enlarged and used as an 8 ½” width flyer. What height must the flyer be?

In problems 184 – 188, solve each percent question. Round decimal answers to the nearest hundredth.

184. A wedding photographer is giving her customers a 15% discount for purchasing photo packages over $250 in the month of July. If a customer in July purchased a package for $525, how much discount did the customer receive?

185. A photographer realizes that he sells on average 85% of the pictures that he develops. If he sells 1224 photos in a month, how many did he develop?

186. A wedding photographer is giving her customers a 20% discount for referring her photography business to a friend. If a customer purchased a package worth $895 and received the discount for a referral, what was the discount price of the package?

187. A graphic design company gives an invoice to a customer in the amount of $357.99. $124 of this amount is considered profit. What is the company’s percent profit for this transaction?

188. A graphic designer receives a 4 ½% raise increasing her annual salary to $47808.75. What was her former annual salary?

7. APPLICATIONS FOR INTEGRATED ENERGY TECHNOLOGY

FUN FACT: To calculate the amount of heat in BTU’s (British Thermal Unit of energy) that is needed to provide the hot water for the length of an average shower, we need to know the flow rate of the shower head in gallons per minute (GPM).

In problems 189 – 191, convert each to unit rates in gallons per minute (GPM).

189. An out-of-date shower head flows at a rate of 20 gallons in 5 minutes. What is the GPM?

190. An out-of-date shower head flows at a rate of 42 gallons in 8 minutes. What is the GPM?

191. When comparing two “low-flow” shower heads, you find that shower head A flows at a rate of 1.125 gallons in 40 seconds and shower head B flows at a rate of 9 gallons in $\frac{2}{3}$ minutes. Which of the two shower heads has the lower GPM?

FUN FACT: To approximate the array size of a solar PV (photovoltaic) system, we need to know (on average) how many kilowatt-hours per day of solar insulation is received in the city in which we live and how many kilowatt-hours per day is used at our home.
In problems 192 – 195, convert each to unit rates in kilowatt-hours (kW-h) per day.

192. On average, San Francisco, California receives 31.5 kilowatt-hours of solar insulation per week. What is the kW-h of solar insulation per day?
193. On average, Rifle, Colorado receives 165 kilowatt-hours of solar insulation per month (30 days = 1 month). What is the kW-h of solar insulation per day?
194. On average, Seattle, Washington receives 912.5 kilowatt-hours of solar insulation per year (365 days = 1 year). What is the kW-h of solar insulation per day?
195. On average, Brooklyn, New York receives 0.125 kilowatt-hours of solar insulation per hour. What is the kW-h of solar insulation per day?

In problems 196 – 199, solve the problem by setting up and solving a proportion. Show your work. Round decimal answers to the nearest hundredth.

196. If a faucet can fill a 5 gallon bucket in 2 minutes, how long would it take the same faucet to fill a 44 gallon bath tub?
197. If a faucet can fill a 1.125 gallon bucket in 35 seconds, how long would it take the same faucet to fill a 60 gallon bath tub?
198. If a city receives 30.1 kWh of solar insulation in 7 days, how many kW-h of solar insulation does the city receive in 66.5 days?
199. If a city receives 7 kWh of solar insulation in 24 hours, how many kW-h of solar insulation does the city receive in 390 hours?

In problems 200 – 205, solve each percent question. Round decimal answers to the nearest hundredth.

200. By installing a solar PV system you receive a 30% federal tax rebate of the cost of the system. If it costs $56,900 to install your solar PV system, what is your federal tax rebate?
201. By installing a solar PV system you receive a 30% federal tax rebate of the cost of the system. If you install a solar PV system and you receive a rebate of $12,300 on your federal taxes, how much was the cost of the solar PV system?
202. In any solar PV system, there are losses in efficiency over time. If you plan to purchase a solar PV system that is 77% efficient and you want your system to be able to have enough energy for 4.4 kW, what array size in kW should you purchase?
203. In any solar PV system, there are losses in efficiency over time. If you plan to purchase a solar PV system that is 77% efficient and you want your system to be able to have enough energy for 5.2 kW, what array size in kW should you purchase?
204. By purchasing a solar PV system, you were able to decrease your electric bill from $324 to $183 per month. What is the percent decrease?
205. By purchasing a solar PV system, you were able to decrease your electric bill from $272 to $50 per month. What is the percent decrease?
8. APPLICATIONS FOR PROCESS TECHNOLOGY

FUN FACT: An oil industry technician will need to work with pressure rates such as PSI (pounds per square inch), PSIA (pounds per square inch absolute), PSIG (pounds per square inch gauge), and flow rates such as gpm (gallons per minute).

In problems 206 – 208, calculate the flow rate as a unit rate in gallons per minute. Round your answer to the nearest hundredth of a gallon per minute.

206. It takes 3 minutes to fill a 42 gallon barrel of crude oil. What is the flow rate?
207. It takes 1 minutes and 40 seconds to fill a 42 gallon barrel of crude oil. What is the flow rate?
208. It takes 5 ½ minutes to fill three 42 gallon barrels of crude oil. What is the flow rate?

In problems 209 – 212, calculate the air pressure as a unit rate in pounds per square inch.

209. The air pressure at sea level is 147 pounds to 10 square inches. What is the air pressure at sea level?
210. The air pressure at an elevation of 5000 feet is 61 pounds to 5 square inches. What is the air pressure at an elevation of 5000 feet?
211. The air pressure at an elevation of 9000 feet is 2.625 pounds to ¼ square inch. What is the air pressure at an elevation of 9000 feet?
212. The air pressure at an elevation of -4500 feet (4500 feet below sea level) is 11.18 pounds to 0.65 square inches. What is the air pressure at an elevation of -4500 feet?

FUN FACT: The slope of a piezometric surface (see figures 1 and 2 on the next page) is called the **hydraulic gradient**. The hydraulic gradient is the ratio of the “change in head” to the “horizontal distance” and it is measured in the direction of the steepest slope of the piezometric surface. Groundwater flows in the direction of the hydraulic gradient and at a rate that is proportional to the slope. Note that the hydraulic gradient has no units since the change in head and the horizontal distance carry the same units (feet, yards, inches, meters, etc.) which cancel when dividing. In Module III we will consider three wells and draw a line between the wells with the highest and lowest head, then divide that distance into equal parts. In Module VI we will determine the direction of the flow of groundwater, the horizontal distance from the well with the highest head to the equipotential line (which we will define later), and finally, the hydraulic gradient. In the next few exercises you will be given the change in head and the horizontal distance between two points that is needed to calculate the hydraulic gradient.
In problems 213 – 215, calculate the hydraulic gradient as a ratio in simplest form.

213. Between two points, the head loss is 0.6 meters and the horizontal distance is 315 meters. Find the hydraulic gradient.

214. Between two points, the head loss is 12.8 feet and the horizontal distance is 1000 feet. Find the hydraulic gradient.

215. Between two points, the head loss is 24 ½ feet and the horizontal distance is 1345 ¾ feet. Find the hydraulic gradient.
FUN FACT: After *fractional distillation* (the process of separating the different components of crude oil), the fractions are treated and blended to get various petroleum products such as gasoline, lubricating oils, kerosene, jet fuel, diesel fuel, heating oil, and chemicals of various grades for making plastics and other polymers. A 42 gallon barrel of crude oil yields approximately 45 gallons of petroleum products after this process is complete (the reason for the increase is analogous to what happens to popcorn after it’s been popped). The diagram given below approximates the amount of gallons (out of 45 gallons) of each different petroleum product that a 42 gallon barrel of crude oil produces.

In problems 216 – 218, solve the problem by setting up and solving a proportion. Show your work. Round decimal answers to the nearest hundredth.

216. If a 42 gallon barrel of crude oil produces 10 gallons of diesel fuel, how many gallons of diesel fuel will 74 gallons of crude oil produce?
217. If a 42 gallon barrel of crude oil produces 4 gallons of jet fuel, how many gallons of jet fuel will 985.7 gallons of crude oil produce?
218. If a 42 gallon barrel of crude oil produces 19 gallons of gasoline, how many gallons of gasoline will 680 ¼ gallons of crude oil produce?

In problems 219 – 224, solve each percent question. Round decimal answers to the nearest hundredth.

219. If 7% of a 45 gallon barrel is jet fuel, how many gallons of jet fuel are present?
220. If at an oil refinery gasoline accounts for 32% of all of its petroleum products, how many total gallons are needed to produce 1,080,000 gallons of gasoline?
221. 10 gallons of diesel fuel is present in a 45 gallon barrel. What percent of the barrel is diesel fuel?
222. 7 out of a 45 gallon barrel is the ratio of other petroleum products produced in an oil refinery. What percent of a 45 gallon barrel is other petroleum products?

223. A 42 gallon barrel of crude oil produces approximately 45 gallons of petroleum products. What percent increase is this?

224. a) If the price for crude oil increased from $22 per barrel to $25 per barrel, what is the percent increase?
   b) If the price for crude oil decreased from $25 per barrel to $22 per barrel, what is the percent decrease?
   c) Explain why the percent increase in part (a) is not the same as the percent decrease in part (b).

FUN FACT: When constructing tanks that are used in oil refineries, engineers must account for an increase in volume in the tank due to the heating process. These next few questions will address this type of increase in volume.

In problems 225 – 227, solve each percent increase question. Round decimal answers to the nearest hundredth.

225. If a volume of 350 gallons will increase by 12.5%, what size tank should be constructed?

226. If a volume of 575 gallons is flowing through a tank that needs to be constructed to account for a 7% increase in volume. What size tank should be constructed?

227. If a 270 gallon tank was constructed to account for an increase in volume in the amount of 10%, what is the maximum volume that this tank can hold before the volume increases due to heating?

9. APPLICATIONS FOR SKI AREA OPERATIONS

FUN FACT: The need for heavy equipment and operators is prevalent at ski areas. Dirt needs to be moved frequently at ski areas to create ponds for water storage and to build ramps, etc. An average bucket loader has a 1 cubic yard (heaped) bucket capacity and the machine operator typically operates at a 15 second digging cycle (i.e., it takes the operator 15 seconds to dig a cubic yard hole).

In problems 228 – 230, calculate the digging rate as a unit rate in cubic yards per minute. Round your answer to the nearest hundredth.

228. It takes an operator 25 minutes to dig 82 cubic yards of dirt. What is the digging rate?

229. An operator can dig 47 cubic yards of dirt in 8 minutes. What is the digging rate?

230. It takes an operator 3.75 minutes to dig 5 cubic yards of dirt.
   a) What is the digging rate?
   b) Assuming it takes on average 15 seconds to dig one cubic yard of dirt, is this machine operator digging at an acceptable rate?
In problems 231 – 232, solve the problem by setting up and solving a proportion. Show your work. Round decimal answers to the nearest hundredth.

231. If it takes a machine operator 4.9 hours to dig 2370 cubic yards of dirt, how long will it take the same machine operator to dig 5000 cubic yards of dirt?
232. If it takes a machine operator 3.5 hours to dig 1950 cubic yards of dirt, how many cubic yards of dirt will the same machine operator dig in 8 hours?

FUN FACT: At ski areas, ponds are filled with water in the off-season so that ski area technicians can use that water during the winter to make snow. In the following problems we will solve problems involving flow rates and the costs associated with making snow.

In problems 233 – 234, solve the problem by setting up and solving a proportion. Show your work. Round decimal answers to the nearest hundredth.

233. a) Suppose that a pond is being filled at a rate of 1800 gallons per hour. How long would it take to fill a 11,625,000 gallon pond?
   b) If there are 24 hours in a day, how many days would it take to fill the pond?
   c) Suppose it takes 1 kilowatt (kW) of power to pump 300 gallons into the pond. How many kW of power is needed to fill the pond?
   d) If it costs $1.20/kW, what would it cost to fill the pond?
234. a) Suppose that a pond is being filled at a rate of 16800 gallons in 8 hours. How long would it take to fill a 10,500,000 gallon pond?
   b) Suppose it takes 1 kilowatt (kW) of power to pump 300 gallons. How many kW of power is needed to fill the pond?
   c) If it costs $1.32/kW, what would it cost to fill the pond?

FUN FACT: Skiers per acre is a rate used to measure the theoretical or actual number of skiers that would be found on a particular trail at any given time. This will of course vary based on the time of day, steepness and surface conditions, trail location and ski area traffic patterns, as well as geographic location of the ski area and skier market variations. Theoretical skiers per acre at a western Colorado ski area for beginner, intermediate, and advance terrain are 35, 15, and 5, respectively.

In problems 235 – 237, calculate the skiers per acre as a unit rate.

235. Suppose that at a ski resort there are 172 skiers on 3 acres of beginner runs. How many skiers per acre are there on the beginner runs?
236. Suppose that at a ski resort there are 8 acres of intermediate runs in which it is estimated to have 124 skiers. How many skiers per acre are there on the intermediate runs?
237. If on 2.5 acres of advanced runs it is determined that there are 17 skiers. How many skiers per acre are there on the advanced runs?
FUN FACT: *Trail mix* is the percentage of skiing terrain at a particular ski area divided into the various classifications of skier ability. The Forest Service long ago decreed that the “ideal” trail mix at a ski area was 20% beginner, 60% intermediate, and 20% advanced.

**In problems 238 – 242, solve each percent question. Round decimal answers to the nearest hundredth.**

238. Suppose that a ski resort has 150 trails. If 60% of those trails are intermediate, how many trails are intermediate?
239. Suppose that 16 trails at a ski resort are advanced. If this number represents 20% of all the trails at this resort, how many trails does this resort have?
240. Suppose that 34 trails out of 156 trails are beginner runs. What percent of the trails are beginner runs?
241. If a ski resort decided to increase the number of advanced trails from 15 to 19, what is the percent increase?
242. If a ski resort decided to decrease the number of intermediate trails from 86 to 80, what is the percent decrease?

FUN FACT: Base lodge capacities can vary greatly based on the ski area location, clientele, etc. In general, the rule of thumb is to provide for between 20% and 35% of the skier capacity in the base lodge facility as well as allowing for 10 square feet of lodge floor space for each skier. In addition, one must properly plan for the correct restroom capacity. General guidelines would suggest one toilet for every 50 to 60 persons.

**In problems 243 – 244, solve each question. Round decimal answers to the nearest hundredth.**

243. Suppose that a ski resort expects 2750 skiers in a day.
   a) If 24% of these skiers are in the Lodge, how many skiers are in the lodge?
   b) If the lodge facility requires 10 square feet of lodge floor space for each skier, what size lodge would be needed to accommodate the skiers?
   c) If there needs to be one toilet for every 60 skiers in the lodge, how many toilets are needed in the lodge?
244. Suppose that a ski resort expects 6400 skiers in a day.
   a) If 40% of these skiers are in the lodge, how many skiers are in the lodge?
   b) If the lodge facility requires 8 square feet of lodge floor space for each skier, what size lodge would be needed to accommodate the skiers?
   c) If there needs to be one toilet for every 80 skiers in the lodge, how many toilets are needed in the lodge?
Solutions to Module II:

1. 1:4  2. 2 to 3  3. 8/5  4. 3:1  5. 1 to 4  6. 2/1  7. 50:127  8. 1 to 4  9. 1 to 4  10. 1 to 1
11. 5 to 8  12. 5 to 12  13. x = 3.5  14. x = 2.98  15. x = 3.94  16. x = 9.33  17. 0.45  18. 0.37
19. 0.005  20. 0.00375  21. \( \frac{3}{10} \)  22. \( \frac{1}{6} \)  23. \( \frac{1}{8} \)  24. \( \frac{19}{4} \)  25. 65.125%  26. 0.37%
27. 102.75%  28. 102.75%  29. 14%  30. 162.5%  31. 65%  32. \( \frac{3}{266} \)  33. 289.68  34. 37.24
35. 29.27  36. 9054.05  37. 92.33%  38. 315.79%  39. $52,600  40. $32,500  41. $128.50  42. 15.01%
43. 41.2%  44. 33 1/3%  45. 54.94%  46. 48%  47. 31.5%  48a. $1078.95  48b. $10,421.05
49. 325 mg/tablet  50. 500 mg/tablet  51. 0.05 g/mL  52. 4 mEq/mL  53. 0.1%  54. 2.5%
55. 0.3%  56. 0.03%  57. 50 gm  58. 50 gm  59. 9 gm  60. 12.5 gm  61. 12.5 gm  62. 1.5 tablets
63. 10 mL  64. 10 mL  65. 0.35 mL  66. Approximately 4 tablets  67. 5 mL  68. 6 mL  69. 1.25 mL
70. 10 mL  71. 3 tablets  72. 4 tablets  73. 3/5 or 0.6 doses  74. 63 pills  75. 2.5 gpm  76. 27.27 gpm
77. 0.12 gpm  78. 6 to 5 79. 17 to 6 80. 2.44 gallons of gas  81. 0.06 gallons of oil  82. 18 mph
83. 6 mph  84. 19% of the foam was left  85. 81% of the foam was used  86. 2.5 hours  87. 72 ignitions.
88. 36 ignitions. Expect 36 ignitions out of 80 glowing firebrads  89. Probability of ignition is 64%
90. 126 glowing firebrads  91. Approximately 0.15; A debt of $0.15 for every $1 of income 92. Approximately 0.21; A debt of $0.21 for every $1 of income 93. Approximately 0.34; $0.34 is paid for mortgage for every $1 of income 94. front-end ratio
is approximately 0.27; back-end ratio is approximately 0.39; The applicant would most likely not qualify 95. front-end ratio is approximately 0.33; back-end ratio is approximately 0.41; The applicant would most likely not qualify 96. back-end ratio is approximately 0.396; The applicant would most likely qualify 97. front-end ratio is approximately 0.22; back-end ratio is approximately 0.32; The applicant would most likely qualify 98. $2744 99. $942.50
100. $2718  101. $861  102. $1722  103. 2.5 customers/seat  104. 3.91 customers/seat
105. 0.81 customers/seat  106. 2.57 customers/seat  107. $0.03/ounce  108. $0.07/ounce
109. $0.02 (or approximately $0.03) per ounce; $0.32 110. 17.5 oz. 111. 350 oz.
112. 7.875 lb. 113. 14 oz. 114. 12.25 c.  115. 1400 mL 116. $0.766, $0.264, $1.03
117. 26.1%  118. $3.00  119. $10.00  120. 22%  121. 30%  122. 33 1/3%  123. 35.3%  124. 10%  125. 40%
126. 24.8%  127. 31.8%  128. 21.3%  129. 9.4%  130. 36.4%  131. 38.5%  132. 22.3%  133. 30.9%  134. 20%
135. $400; $11000 136. $1000; $2600  137. $1200; $5385  138. $2380; $7700 139. $2400; $11962.50
140. $10,980; $38,647.50  141. 28.41%  142. 35%  143. 1400 lbs.  144. 440 lbs.  145. 560 lbs.  146. 323 lbs.
147. 360; 158.8 lbs.  148. $0.38/ounce  149. $960.56, $2475, 38.8%; Th: 59.9%, $2767.41, $5490, 50.4%; Fri:
37.7%, $4772.56, $10,804, 44.2%; Sat: 22.4%, $6131.11, $16,881, $36.3%  149. 34.375%, 138, 125
150. 43.75%, 175, 147 151. 21.875%, 87, 87 152. a) 75.7%  b) 24.3%  c) $1,850,000  d) $434,000
153. $99,000 f) $533,000; 28.8% g) $1,317,000, 71.2% h) 24.9% i) 5.1% j) $37,000 k) $129,500 l) 0.7% m) $55,500
n) $51,800 o) 1.8% p) 1.1% q) $895,800, 48.4% r) $421,200; 22.8% s) 16.2% t) $121,200; 6.6% u)
$77,700 v) $43,500; 2.4% 153. 58:100 154. 46% 155. \( \frac{3}{6} \) 156. \( \frac{3}{21} = \frac{13}{91} \) 157. 2:3 158. \( \frac{13}{3} \) 159. 13 to 17 160. 34% 161. 0.55 162. 38% 163. 0.375 164.
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>165. 50</td>
<td>166. 1/6</td>
</tr>
<tr>
<td>175. 1800 DPI</td>
<td>176. 300 PPI</td>
</tr>
<tr>
<td>182. 2.25 inches</td>
<td>183. 1.0” height</td>
</tr>
<tr>
<td>191. Shower head A</td>
<td>192. 4.5 kW-h/day</td>
</tr>
<tr>
<td>200. $17,070</td>
<td>201. $41,000</td>
</tr>
<tr>
<td>211. 10.5 psi</td>
<td>212. 17.2 psi</td>
</tr>
<tr>
<td>221. 22.22%</td>
<td>222. 15.56%</td>
</tr>
<tr>
<td>229. 5.88 cubic yards/min</td>
<td>230. a) 1.33 cubic yards/min</td>
</tr>
<tr>
<td>c) $46,200</td>
<td>235. 57.33</td>
</tr>
<tr>
<td>244. a) 2560</td>
<td>b) 20,480 square feet</td>
</tr>
</tbody>
</table>
Module III
Units of Measure

Because units of length, volume, weight, area, temperature, energy, and money differ around the world, it is important to understand how to convert from one unit to the next. Emergency Medical Technicians and nurses need to convert between the metric system and U.S. customary system to accurately calculate dosages; accountants and bookkeepers may need to convert monetary value by using the current exchange rates; firefighters would need to convert unfamiliar units to units that are familiar to them for sound planning; professional chefs convert units of ingredients to help calculate costs; early childhood educators teach children how units vary in size and shape; graphic designers convert U.S. customary lengths to points and picas to measure type lines, images, and pages; photographers convert millimeters to inches to help determine the bellows factor when adjusting the f-stop; solar photovoltaic installers convert kilowatts to watts when calculating the total cost of a solar PV system; oil industry technicians need to convert units of pressure, flow, temperature, density, and energy; and ski area managers need to convert units of volume, area and time to help manage costs and maintain a desirable comfort level for ski patrons.

The practice examples in this section require the use of a conversion table (Table 3.1) that provides us with the necessary information to convert lengths, volumes, weight, area, or temperature within a) the U.S. customary system (measurement system adopted by the United States), b) the metric system (base 10 measurement system that is used by most of the world), and c) converting between the U.S. customary and metric system. The homework problems, however, may require additional conversion factors outside of what is given in the conversion table, depending on the area of emphasis. Any conversion factor that is not included in Table 3.1 and is needed to solve a conversion problem will be provided with the homework problem.

I. U.S. Customary Measurement System

Khan Academy Resources: https://www.khanacademy.org/math/5th-engage-ny/engage-5th-module-2/5th-module-2-topic-d/e/converting-units--us-customary-

In this section we’ll discuss four measurement groups in the U.S. customary system:

1. Linear (or length)
2. Liquid (or volume)
3. weight (or mass)
4. Area

Our Goal: We want to be able to convert from one unit of measure to another within the U.S. customary system, such as converting inches to feet. The process that we will use to do this is
known as **dimensional analysis** (or **unit analysis**). A detailed “recipe” for this process is given on the next page.

**How To Convert Units Using Dimensional Analysis:**
1. Write down the given unit of measurement as a fraction (include units).
2. Determine what unit of measurement you wish to convert to.
3. Find the unit factor (or conversion factor) on Table 3.1 that connects the “undesired” unit (unit that you are converting from) with the “desired” unit (unit you are converting to). You may need to use more than one unit factor to make the connection.
4. Multiply the “undesired” unit by a series of unit factors to cancel out the given unit and achieve the “desired” unit. To cancel identical units, one of the units must appear in the numerator and the other must appear in the denominator.

Note: A **unit factor** is a ratio that compares the equality of two differing units. For example, 12 in. = 1 ft translates to the following unit factors: \( \frac{12 \text{ in.}}{1 \text{ ft}} \) or \( \frac{1 \text{ ft}}{12 \text{ in.}} \). We will use Table 3.1 given below to identify unit factors that will help us convert units within the U.S. customary system AND between the U.S. customary system and the metric system.

### Table 3.1: Table of Conversions

<table>
<thead>
<tr>
<th>Units of length:</th>
<th>(U.S. Cust. to Metric)</th>
<th>(Metric to U.S. Cust.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ft (foot) = 12 in. (inches)</td>
<td>1 in. ≈ 2.54 cm (centimeter)</td>
<td>1 m (meter) ≈ 3.3 ft</td>
</tr>
<tr>
<td>1 yd (yard) = 3 ft</td>
<td>1 yd ≈ 0.914 m</td>
<td>1 m ≈ 1.094 yd</td>
</tr>
<tr>
<td>1 mi (mile) = 5280 ft</td>
<td>1 mi ≈ 1.609 km (kilometer)</td>
<td>1 km ≈ 0.621 mi</td>
</tr>
</tbody>
</table>

**Units of capacity:**
- 1 c (cup) = 8 fl-oz (fluid ounces)  
- 1 pt (pint) = 2 c  
- 1 qt (quart) = 2 pt  
- 1 gal (gallon) = 4 qt

**Units of weight:**
- 1 lb (pound) = 16 oz (ounces)  
- 1 T (U.S. ton) = 2000 lb  
- 1 kg (kilogram) = 0.454 kg (kilogram)  
- 1 kg ≈ 2.204 lb

**Units of Area:**
- 1 \( \text{mi}^2 \) (square mi) = 27,878,400 \( \text{ft}^2 \) (square ft)  
- 1 \( \text{yd}^2 \) (square yd) = 9 \( \text{ft}^2 \)  
- 1 \( \text{ft}^2 \) = 144 \( \text{in.}^2 \) (square in.)  
- 1 acre ≈ 43,560 \( \text{ft}^2 \)

**Units of time:**
- 1 yr (year) = 365 ¼ d  
- 1 d (day) = 24 hr  
- 1 hr (hour) = 60 min  
- 1 min (minute) = 60 sec
Units of temperature:

Degrees Fahrenheit (°F): \[ F = \frac{9}{5}C + 32 \]

Degrees Celsius (°C): \[ C = \frac{5}{9}(F - 32) \]

Example 1: Use dimensional analysis to convert 2.34 yd to ft.

Solution to Example 1:

1. Write down the given unit of measurement as a fraction (include units):
\[ \frac{2.34 \text{ yd}}{1} \]

2. Determine what unit of measurement you wish to convert to: We want to convert to feet.

3. Find the unit factor in Table 3.1 that connects the “undesired” unit (i.e. yard) with the “desired” unit (i.e. feet): 1 yd (yard) = 3 ft

4. Multiply the “undesired” unit by the unit factor to cancel out the given unit and achieve the “desired” unit:
\[ \frac{2.34 \text{ yd}}{1} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} = 7.02 \text{ ft} \]. Answer: 7.02 ft

Example 2: Use dimensional analysis to convert \( \frac{3}{4} \) gal to fl-oz.

Solution to Example 2:

1. Write down the given unit of measurement as a fraction (include units):
\[ \frac{1 \frac{3}{4} \text{ gal}}{1} \]

2. Determine what unit of measurement you wish to convert to: We want to convert to fluid ounces.

3. Find the unit factor in Table 3.1 that connects the “undesired” unit (i.e. gallons) with the “desired” unit (i.e. fluid ounces): In this case, we do not have a unit factor that connects gallons directly to fluid ounces, so we will need to use a series of unit factors.

4. Multiply the “undesired” unit by the series of unit factors to cancel out the given unit and achieve the “desired” unit:
\[ \frac{1 \frac{3}{4} \text{ gal}}{1} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} \cdot \frac{2 \text{ pt}}{1 \text{ qt}} \cdot \frac{2 \text{ fl-oz}}{1 \text{ pt}} = 224 \text{ fl-oz} \]. Answer: 224 fl-oz
Example 3: Convert $18\frac{2}{3}$ ounces to pounds.

Solution to Example 3:
1. Write down the given unit of measurement as a fraction (include units):
   \[
   \frac{18\frac{2}{3} \text{ oz}}{1}
   \]
2. Determine what unit of measurement you wish to convert to: We want to convert to pounds.
3. Find the unit factor in Table 3.1 that connects the “undesired” unit (i.e. ounces) with the “desired” unit (i.e. pounds): \(1 \text{ lb} (\text{pound}) = 16 \text{ oz} (\text{ounces})\)
4. Multiply the “undesired” unit by the unit factor to cancel out the given unit and achieve the “desired” unit:
   \[
   \frac{18\frac{2}{3} \text{ oz}}{1} \cdot \frac{1 \text{ lb}}{16 \text{ oz}} = \frac{1\frac{1}{6}}{1} \text{ lb} \approx 1.17 \text{ lb} \quad \text{Answer: 1.17 lb}
   \]

Example 4: CMC services an area of approximately 12,000 square miles. Use dimensional analysis to approximate how many acres are in 12,000 square miles?

Solution to Example 4:
1. Write down the given unit of measurement as a fraction (include units):
   \[
   \frac{12,000 \text{ mi}^2}{1}
   \]
2. Determine what unit of measurement you wish to convert to: We want to convert to acres.
3. Find the unit factor in Table 3.1 that connects the “undesired” unit (i.e. square miles) with the “desired” unit (i.e. acres): In this case, we do not have a unit factor that connects square miles directly to acres, so we will need to use a series of unit factors.
4. Multiply the “undesired” unit by the series of unit factors to cancel out the given unit and achieve the “desired” unit:
   \[
   \frac{12,000 \text{ mi}^2}{1} \cdot \frac{27,878,400 \text{ ft}^2}{1 \text{ mi}^2} \cdot \frac{1 \text{ acre}}{43,560 \text{ ft}^2} = 7,680,000 \text{ acres}
   \]
   
   Answer: 7,680,000 acres
II. Metric Measurement System

Khan Academy Resources: [https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-measurement-topic/cc-5th-unit-conversion/v/unit-conversion](https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-measurement-topic/cc-5th-unit-conversion/v/unit-conversion)

In this section we’ll discuss four measurement groups in the metric system:

1. Linear (or length)
2. Liquid (or volume)
3. Weight (or mass)
4. Area

Our Goal: We want to be able to convert from one unit of measure to another within the metric system, such as converting centimeters to meters. The major difference between the metric system and the U.S. customary system is that we will not have to use dimensional analysis to convert from one metric unit to the other. This is due to the fact that the metric system operates like our number system, which is a base 10 system, meaning that each digit can be written as an integral power of ten. Each unit in the metric system is named relative to a base unit (or basic unit), so when converting from one unit to another in the metric system, it is simply a decimal shift from the original number to the converted number.

In the metric system the following are base units:

<table>
<thead>
<tr>
<th>Type of Measurement</th>
<th>Base Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear or Length</td>
<td>meter (m)</td>
</tr>
<tr>
<td>Liquid or Volume</td>
<td>liter (L)</td>
</tr>
<tr>
<td>Weight or Mass</td>
<td>gram (g)</td>
</tr>
</tbody>
</table>

The following table (Table 3.2) lists the base unit and some other metric units relative to the base unit:

<table>
<thead>
<tr>
<th>Unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>kilo (k)</td>
<td>hecto (h)</td>
</tr>
<tr>
<td>kilometer</td>
<td>km</td>
</tr>
<tr>
<td>kiloliter</td>
<td>kL</td>
</tr>
<tr>
<td>kilogram</td>
<td>kg</td>
</tr>
</tbody>
</table>

Note: 1 metric ton (t) = 1000 kg and 1 cubic centimeter (cc) = 1 mL
A detailed “recipe” for converting within the metric system is given on the next page.

**Converting metric units to different metric units:**
1. Use Table 3.2 to find the metric unit that you wish to convert.
2. Count the number of units and note the direction (right or left) of the movement from the “undesired” unit to the “desired” unit of measure.
3. Move the decimal point in the given measurement the same number of places and in the same direction to obtain the converted measurement.

Note: If you are converting between metric units of area (square units), we can use this same process, except we **double** the number of decimal movements. If we are converting between metric units of volume (cubic units) then we can **triple** the number of decimal movements.

**Example 5:** Convert 0.05 hL to liters.

Solution to Example 5:
1. Use Table 3.2 to find the metric unit that you wish to convert:

<table>
<thead>
<tr>
<th>kiloliter</th>
<th>hectoliter</th>
<th>dekaliter</th>
<th>liter</th>
</tr>
</thead>
<tbody>
<tr>
<td>kL</td>
<td>hL</td>
<td>daL</td>
<td>L</td>
</tr>
</tbody>
</table>

   “Undesired Unit”                             “Desired Unit”

2. Count the number of units and note the direction (right or left) of the movement from the “undesired” unit to the “desired” unit of measure: Move the decimal 2 places to the right to go from hL to L.

3. 0.05 hL = 5 L. **Answer:** 5 L

**Example 6:** Convert 400 cc’s to liters.

Solution to Example 6:
In this example we must know that 1 cc = 1 mL. We can think that we are converting 400 mL to liters.

1. Use Table 3.2 to find the metric unit that you wish to convert:

<table>
<thead>
<tr>
<th>Liter</th>
<th>deciliter</th>
<th>centiliter</th>
<th>milliliter</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>dL</td>
<td>cL</td>
<td>mL</td>
</tr>
</tbody>
</table>

   “Desired Unit”                             “Undesired Unit”

2. Count the number of units and note the direction (right or left) of the movement from the “undesired” unit to the “desired” unit of measure: Move the decimal 3 places to the left to go from mL to L.
3. 400 cc’s = 400 mL = 0.4 L. Answer: 0.4 L

Example 7: Convert $580,000 \text{ m}^2$ to square kilometers.

Solution to Example 7:
1. Use Table 3.2 to find the metric unit that you wish to convert:

<table>
<thead>
<tr>
<th>kilometer</th>
<th>hectometer</th>
<th>dekameter</th>
<th>meter</th>
</tr>
</thead>
<tbody>
<tr>
<td>km</td>
<td>hm</td>
<td>dam</td>
<td>m</td>
</tr>
</tbody>
</table>

“Desired Unit” “Undesired Unit”

2. Count the number of units and note the direction (right or left) of the movement from the “undesired” unit to the “desired” unit of measure: Move the decimal 3 places to the left to go from m to km:

<table>
<thead>
<tr>
<th>kilometer</th>
<th>hectometer</th>
<th>dekameter</th>
<th>meter</th>
</tr>
</thead>
<tbody>
<tr>
<td>km</td>
<td>hm</td>
<td>dam</td>
<td>m</td>
</tr>
</tbody>
</table>

3. Since we are converting square meters to square kilometers, we must DOUBLE the number of decimal movements. Therefore, we should move the decimal $3 \times 2 = 6$ places to the left: $580,000 \text{ m}^2 = 0.58 \text{ km}^2$. Answer: 0.58 km$^2$

III. Metric $\leftrightarrow$ U.S. Customary Conversions

To convert between the metric system and U.S. customary system we must use dimensional analysis and Table 3.1 given in part I. It is helpful to note whether we are converting from the U.S. customary system to metric or metric to U.S. customary when choosing which unit factor to use.

Example 8: Convert $\frac{3}{8}$ in. to mm.

Solution to Example 8:
1. Write down the given unit of measurement as a fraction (include units):
   $$\frac{3/8 \text{ in.}}{1}$$

2. Determine what unit of measurement you wish to convert to: We want to convert to millimeters.
3. Find the unit factor in Table 3.1 that connects the “undesired” unit (i.e. inches) with the “desired” unit (i.e. millimeters): In this example Table 3.1 does not have a unit factor that directly connects inches to millimeters, however, if we can bridge over from U.S. customary to the metric system we will be able to convert. We will use \(1 \text{ in.} \approx 2.54 \text{ cm (centimeter)}\) to convert to the metric system.

4. Multiply the “undesired” unit by the unit factor to cancel out the given unit and achieve the “desired” unit:

\[
\frac{3/8 \text{ in.}}{1} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 0.9525 \text{ cm} = 9.525 \text{ mm}.
\]

Answer: 9.525 mm

Example 9: Convert 40 mL to fluid ounces.

Solution to Example 9:
1. Write down the given unit of measurement as a fraction (include units):
\[
\frac{40 \text{ mL}}{1}
\]
2. Determine what unit of measurement you wish to convert to: We want to convert to fluid ounces.
3. Find the unit factor in Table 3.1 that connects the “undesired” unit (i.e. milliliters) with the “desired” unit (i.e. fluid ounces): In this example Table 3.1 does not have a unit factor that directly connects milliliters to fluid ounces so we will use \(1 \text{ L} \approx 1.057 \text{ qt}\) to convert to the U.S. customary system, then continue to use unit factors to achieve the “desired” unit.
4. Multiply the “undesired” unit by the unit factors to cancel out the given unit and achieve the “desired” unit:

\[
40 \text{ mL} = 0.04 \text{ L} = \frac{0.04 \text{ L}}{1} \cdot \frac{1.057 \text{ qt}}{1 \text{ L}} \cdot \frac{2 \text{ pt}}{1 \text{ qt}} \cdot \frac{2 \text{ fl-oz}}{1 \text{ pt}} \approx 1.353 \text{ fl-oz}.
\]

Answer: 1.353 fl-oz

Example 10: Convert -25°C to degrees Fahrenheit.

Solution to Example 10:
In this example we need to use the formula for converting temperature. Because we are converting from °C to °F, we will use the formula: \(F = \frac{9}{5}C + 32\). Substitute -25 in for C, then solve for F: \(F = \frac{9}{5}(-25) + 32 = -45 + 32 = -13\). Answer: -13°F
Converting Rates Using Dimensional Analysis:
In some examples we will be asked to convert a given rate to a rate with different units, for example ounces per second to gallons per minute. We may use dimensional analysis to do this.

Example 11: Convert 40 miles per hour to feet per second.

Solution to Example 11:
1. Write down the given unit of measurement as a fraction (include units):
\[
\frac{40 \text{ mi}}{1 \text{ hr}}
\]
2. Determine what unit of measurement you wish to convert to: We want to convert to feet per second.
3. In this example, we need to eliminate two units (miles and hours) and convert to (feet and seconds) we can use Table 3.1 to find the unit factors: 1 mi (mile) = 5280 ft, 1 hr = 60 min, and 1 min = 60 s.
4. Multiply the “undesired” units by the unit factors to cancel out the given unit and achieve the “desired” unit:
\[
\frac{40 \text{ mi}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \approx 58.67 \frac{\text{ft}}{s}.
\]

Answer: \( 58.67 \frac{\text{ft}}{s} \)

Homework Set:
In problems 1 – 5, use dimensional analysis (or unit analysis) to convert the units.
1. 9 ft to inches.
2. 324 oz to pounds.
3. 4 ½ gal to pints.
4. 24 min to seconds.
5. A 12 ft. by 14.5 ft. room is to be carpeted. Calculate the area of the room in square yards.

In problems 6 – 14, use any method to convert the units.
6. 1.2 km to meters.
7. 950 cm to meters.
8. 700 g to kilograms.
9. 0.0085 t to hectograms.
10. \( 25 \text{ dm}^2 \) to square meters.
11. \( 0.3 \text{ m}^2 \) to square centimeters.
12. 45 L to kiloliters.
13. 0.02 L to cc’s.
14. 500 yd to kilometers.
1. APPLICATIONS FOR EMT/MEDICAL ASSISTANT/NURSING

FUN FACT: Three systems of measurement that are used for measuring drugs and solutions today are the metric, apothecary, and household systems. Developed in France in 1799, the metric system is the adopted system for measurements for most countries. The United States currently uses the U.S. customary system, but has steadily been moving toward using only the metric system. The metric system is also known as the International System of Units (SI units).

In problems 15 – 30, convert the metric measurements:

15. 10.5 g to milligrams.
16. 0.5 mg to micrograms. Note: 1 milligram = 1000 micrograms.
17. 45 kilograms to g.
18. 2.5 liters to mL.
19. 3.25 L to mL.
20. 20 cL to mL.
21. 125 decigrams to milligrams.
22. 0.25 kg to g.
23. 500 mg to grams.
24. 7500 micrograms to milligrams. Note: 1 milligram = 1000 micrograms.
25. 350 g to kg.
26. 0.5 mL to L.
27. 325 milligrams to g.
28. 50 mL to deciliters.
29. 2750 milliliters to dL.
30. 85 millimeters to cm.

FUN FACT: First used in England and then introduced to the U.S., the apothecary system is a measurement system that dates back to the Middle Ages. The basic unit of measurement used for weight in the apothecary system is the grain (gr), derived from the weight of a single grain of wheat. Three units that are used for measuring volume in the apothecary system are minim (m), dram (dr) and ounce (oz). In the U.S., the apothecary system is rapidly being phased out since the measurements are considered to be estimates and therefore unreliable.
In problems 31 – 38, use the apothecary measurement table below and dimensional analysis to convert each measurement.

<table>
<thead>
<tr>
<th>Weight Conversions in Apothecary System:</th>
<th>Liquid Conversions in Apothecary System:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 grain (gr) ≈ 60 – 65 mg</td>
<td>1 quart (qt) = 2 pints (pt)</td>
</tr>
<tr>
<td>1 ounce (oz) = 480 grains (gr)</td>
<td>1 pint (pt) = 16 fluid ounces (fl-oz)</td>
</tr>
<tr>
<td>1 ounce (oz) = 8 drams (dr)</td>
<td>1 fluid ounce (fl oz) = 8 fluid drams (fl dr)</td>
</tr>
<tr>
<td>1 dram (dr) = 60 grains (gr)</td>
<td>1 minim (m) = 1 drop (gt)</td>
</tr>
<tr>
<td></td>
<td>Note: drops = gtt</td>
</tr>
</tbody>
</table>

31. 5 dram to gr.
32. 5 fl oz to fl dr.
33. 3 qt to pt.
34. 2 pt to fl oz.
35. 240 gr to dr.
36. 16 dr to fl oz.
37. 24 fl dr to fl oz.
38. 15 m to gtt.

FUN FACT: The *household system* of measurement uses measuring devices that are commonly used for cooking in the home. The household system is the least accurate of the three medical measurement systems when compared to the apothecary and metric systems due to high variances in measuring devices. For example, a household teaspoon can hold anywhere from 4 to 7 mL of fluid.

In problems 39 – 48, use the household measurement table below and dimensional analysis to convert each measurement.

<table>
<thead>
<tr>
<th>Conversions in the Household System</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 drop (gt) = 1 minim (m)</td>
</tr>
<tr>
<td>1 teaspoon (t) = 60 drops (gtt), 5 mL</td>
</tr>
<tr>
<td>1 tablespoon (T) = 3 teaspoons (t)</td>
</tr>
<tr>
<td>1 ounce (oz) = 2 tablespoons (T)</td>
</tr>
<tr>
<td>1 coffee cup = 6 ounces (oz)</td>
</tr>
<tr>
<td>1 medium size glass = 8 ounces (oz)</td>
</tr>
<tr>
<td>1 measuring cup (c) = 8 ounces (oz)</td>
</tr>
</tbody>
</table>
39. 2 glasses to oz.
40. 3 ounces to T.
41. 4 tablespoons to t.
42. 1 ½ coffee cups to oz.
43. ½ teaspoon to gtt.
44. 9 teaspoons to T.
45. 6 tablespoons to oz.
46. 90 drops to t.
47. 12 ounces to c.
48. 24 ounces to medium size glasses.

In problems 49 – 51, use dimensional analysis to answer the following questions.

49. A drug is available in 20 mg/mL. The patient needs 50 mg of the drug. How many mL should be administered?
50. A drug is available in 100 mg/mL. The patient needs 250 mg of the drug. How many mL should be administered?
51. A drug is available in 1200 mg/120 mL. The patient needs 0.75 g of the drug. How many mL should be administered?

In problems 52 – 56, use dimensional analysis to answer the following questions. Round answers less than 1 mL to the hundredth and answers greater than 1 mL to the tenth.

52. A physician orders Digoxin 0.5 mg IVP STAT (“intravenous push medication”, “immediately”) to be administered over 5 minutes. The dose available is Digoxin 0.25 mg/mL. Calculate how many mL the nurse should give every 15 seconds.
53. A physician orders Chlorpromazine hydrochloride 2 mg to be administered over 2 minutes. The dose available is Chlorpromazine hydrochloride 25 mg/mL. Calculate how many mL the nurse should give every 15 seconds.
54. A physician orders Haloperidol Lactate 4 mg IVP to be administered over 1.5 minutes. The dose available is Haloperidol Lactate 5 mg/mL. Calculate how many mL the nurse should give every 15 seconds.
55. A patient is to receive 250 milliliters of 5% D/W (D,W) solution IV over 6 hours. The label on the box of the IV indicates that 15 drops dissipate 1 milliliter of the solution. How many drops should the patient receive each minute?
56. 500 milliliters of 5% D/W solution IV contains 20,000 units of heparin. If the patient receives 10 milliliters per hour, how many units is the patient receiving each hour?

FUN FACT: Some drugs are measured by a special designation called a unit. One such drug, Insulin, is manufactured in solutions that contain 100 units per milliliter and is labeled U-100. Doses of electrolytes, such as potassium, may be measured in milliequivalents (mEq). An equivalent (Eq) is the molecular weight of an ion divided by the number of hydrogen ions it reacts with. A milliequivalent is 1/1000 of an equivalent.
2. APPLICATIONS FOR FIRE SCIENCE

FUN FACT: Problem solving in units that are familiar to fire staff is crucial to sound planning. Making a mistake with units can be very costly in the field.

FUN FACT: In the U.S. the chain (ch) is a measurement of length that is frequently used to describe the rate of spread of wildfires. The rate is given in chains per hour. One chain is roughly 66 feet.

In problems 57 – 65, use the table given below to answer the following questions. Round each answer to the nearest hundredth.

### Additional Conversions for Firefighters

<table>
<thead>
<tr>
<th>Units of Length:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 chain (ch) = 66 feet</td>
</tr>
<tr>
<td>1 mile = 80 chains</td>
</tr>
<tr>
<td><strong>Units of Area:</strong></td>
</tr>
<tr>
<td>1 acre = 10 square chains (ch²)</td>
</tr>
<tr>
<td><strong>Units of Volume:</strong></td>
</tr>
<tr>
<td>1 cubic foot (ft³) = 7.48052 gallons (gal)</td>
</tr>
<tr>
<td>1 gal = 0.003485 cubic meters (m³)</td>
</tr>
<tr>
<td>1 gal = 0.13346 ft³</td>
</tr>
<tr>
<td>1 gal = 231 cubic inches (in³)</td>
</tr>
<tr>
<td><strong>Units of Force:</strong></td>
</tr>
<tr>
<td>1 kg m/s² = 0.2248 pounds</td>
</tr>
<tr>
<td>1 slug ft/ s² = 1 pound</td>
</tr>
<tr>
<td><strong>Units of Pressure:</strong></td>
</tr>
<tr>
<td>1 psi = 2.036 in Hg at 32°F</td>
</tr>
<tr>
<td>1 atm = 14.7 psi</td>
</tr>
<tr>
<td>1 psi = 2.304 ft of water</td>
</tr>
<tr>
<td><strong>Units of Weight:</strong></td>
</tr>
<tr>
<td>1 gallon of water = 8.34 pounds</td>
</tr>
<tr>
<td>1 cubic foot of water = 62.4 pounds</td>
</tr>
</tbody>
</table>

57. How many seconds are in 3 hours and 36 minutes?
58. How many pints are in a 5 gallon pail?
59. How many cups are in a 5 gallon pail?
60. Javier constructed 2,678 feet of dozer line. How many chains of dozer line did he construct?
61. Ludka's crew has been out on the Marre Fire for 2 weeks and 2 days. How many hours have they been there?
62. Ned fills a backpack pump with 5 gallons of water. How much weight in water has he added to his pack?
63. A fire burns an area approximately 150 feet wide and 800 feet long. How many acres burned?
64. Jane needs to fill a chain saw gas tank plus 4 sig bottles with a 33:1 gas/oil mix. A sig gas bottle holds 1 liter. The chain saw gas tank holds 1.2 pints. How much gas and how much oil will be mixed to fill the gas tank and four sig bottles?
65. You have been told that a fire is expected to spread at a rate of 100 feet per minute. What is the rate of fire spread in miles per hour?

FUN FACT: A **map scale** is a scale that is printed in a maps legend. It is given as a ratio of inches on the map corresponding to inches, feet, or miles on the ground. For example, a map scale indicating a ratio of 1:24,000 (in/in), means that for every 1 inch on the map, 24,000 inches have been covered on the ground. Ground distances on maps are usually given in feet or miles.

In problems 66 – 68, answer the following questions.

66. Convert the map scale of 1:24,000 (in/in) to (in/ft).
67. Convert the 1:2,000 (in/ft) to (in/mile).
68. The map distance between two points is 6 inches. The map scale is 1:24,000 (in/in). What is the ground distance in feet?

3. APPLICATIONS FOR ACCOUNTING

In problems 69 – 78, use the exchange rates for July 8th, 2010 to answer the questions. For example, 1 U.S. Dollar = 0.79745 Euros. Round each answer to the nearest hundredth.

| U.S. Dollar | Euro | Japanese Yen | British Pound | Swiss Franc | Canadian Dollar | Australian Dollar | Mexican Peso | Hong Kong Dollar |
|-------------|------|--------------|---------------|-------------|----------------|------------------|--------------|----------------
| 1 U.S. Dollar | 0.79745 | 87.661 | 0.65837 | 1.06566 | 1.06494 | 1.18809 | 13.0853 | 7.79514 |

69. Which is more valuable? 1 British Pound or 1 Australian Dollar.
70. Which is more valuable? 5 Hong Kong Dollars or 2 Euros.
71. Which is more valuable? 15 Canadian Dollars or 1000 Japanese Yen.
72. Which is more valuable? 50 Swiss Franc or 780 Mexican Pesos.
73. Convert 80 Euro to U.S. Dollars.
74. Convert 500 U.S. Dollars to Swiss Franc.
75. Convert 1280 Hong Kong Dollars to British Pounds.
76. Convert 382 Swiss Francs to Euros.
77. Convert 1000 Australian Dollars to Mexican Pesos.
78. Convert 2458 U.S. Dollars to Euros.
4. APPLICATIONS FOR CULINARY ARTS

FUN FACT: As a professional chef it is common to purchase bulk ingredients in pounds; however, the recipe may call for that ingredient in ounces. Being able to make conversions in weight and volume are crucial in culinary arts.

In problems 79 – 93, use the table given below and dimensional analysis to convert.

<table>
<thead>
<tr>
<th>Additional Conversions for Culinary Arts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units of Volume:</td>
</tr>
<tr>
<td>1 tablespoon (tbsp) = 3 teaspoons (tsp)</td>
</tr>
<tr>
<td>1 cup (c) = 16 tablespoons</td>
</tr>
<tr>
<td>Note: The unit “#” means pounds</td>
</tr>
</tbody>
</table>

79. 13 pounds to oz.
80. 3.5 gal to qt.
81. 0.875 pt to c.
82. 24 tsp to cups.
83. 9 cups to quarts.
84. 76 oz to pounds.
85. 1/3 cup to tbsp.
86. 2.5 kg to g.
87. 1750 mL to L.
88. 75 fl oz to gal.
89. 4.0325 # to oz.
90. 105 tbsp to qt.
91. 6 oz to grams.
92. 600 grams to #.
93. ¾ oz to grams.

In problems 94 – 98, convert the mixed units to the designated unit.

94. Convert 5 # 11 oz to kilograms.
95. Convert 16 quarts 3 tbsp to liters.
96. Convert 0.25 gallons 3 cups to quarts.
97. Convert 2 pounds 9 oz to ounces.
98. Convert 3 pt 108 tbsp to gallons.
In problems 99 – 102, use dimensional analysis to answer the following questions.

99. If one cup of grated Parmesan cheese weighs 4 ounces, how many pounds of Parmesan cheese must a chef purchase to have 5 cups of grated cheese?

100. If one cup of peanut butter weighs 9 ounces, how many ounces does 5 tablespoons of peanut butter weigh?

101. How many tablespoons of salt are in a container of salt that weighs 1 pound 10 ounces if 1 tablespoon of salt weighs $\frac{2}{3}$ ounces.

102. The covers-per-server standard at Bill’s restaurant is 25 per hour. The manager projects total covers of 300 for the 3-hour lunch period. How many servers should be scheduled?

5. APPLICATIONS FOR EARLY CHILDHOOD EDUCATION

Note: Early childhood educators should have a firm understanding of how to convert the basic units in the metric system and U.S. customary system. We recommend that problems 1 – 14 be assigned in addition to the following problems.

In problems 103 – 111, round each number to the indicated digit.

103. Round 803 to the nearest hundred.

104. Round 5,457 to the nearest ten.

105. Round 9,553 to the nearest thousand.

106. Round 6,842 to the nearest ten-thousand.

107. Round 3375.1887 to the nearest hundredths.

108. Round 7029.0532 to the nearest tenths.

109. Round 7994.4922 to the nearest thousandths.

110. Round 8394.3215 to the nearest hundred.

111. Round 1,999,999 to the nearest ten.

In problems 112 – 113, convert the following temperatures.

112. Convert 64° F to °C.

113. Convert 93° C to °F.

6. APPLICATIONS FOR GRAPHIC DESIGN/PROFESSIONAL PHOTOGRAPHY

FUN FACT: Graphic designers use a variety of measurements in their field. Basis size is the standard length and width, in inches of a grade of paper. The table given below compares the paper grade and its corresponding basis size. Basis weight is a weight, given in pounds, of a ream (500 sheets) of paper cut to the basic size for its grade. Caliper is the thickness of paper in thousandths of an inch. Bulk is the thickness of paper stock in thousandths of an inch or number of pages per inch.
In problems 114 – 116, answer the following questions.

114. A 750-sheet stack of 60-pound book paper measured 3” thick. How thick was each 60-pound sheet?
115. A 256 page book, excluding covers, is printed on a soft, bulky paper that has a caliper of 0.006 inches. How thick, in inches was the book (excluding covers)?
116. How many pounds does 850 sheets of 20-pound per ream bond paper with a basis size 17” × 22” weigh?

FUN FACT: In photography megapixels and pixel resolution help photographers determine the maximum size of a print. 1 megapixel is equal to 1,000,000 pixels.

In problems 117 – 120, use the conversion factor 1 megapixel = 1,000,000 pixels to convert.

117. A photograph has 15,000,000 pixels. How many megapixels does it have?
118. A photograph has 12,500,000 pixels. How many megapixels does it have?
119. A photograph has 10.25 megapixels. How many pixels does it have?
120. A photograph has 15.05 megapixels. How many pixels does it have?

FUN FACT: Bit depth, also known as tonal resolution, is the number of bits used to represent each pixel. The greater the bit depth, the more colors or grayscales are represented. A bit depth of 24 is equal to 16.7 million colors.

In problems 121 – 124, answer the following questions. Round your answer to the nearest thousandth.

121. A 200 mm camera lens has a focal length that is 200 mm. What is the focal length in inches?
122. A 90 mm camera lens has a focal length that is 90 mm. What is the focal length in inches?
123. A 210 mm camera lens has a focal length that is 210 mm. What is the focal length in inches?
124. A 300 mm camera lens has a focal length that is 300 mm. What is the focal length in inches?

FUN FACT: A byte is a unit of digital data measure comprised of eight (8) binary characters (1’s or 0’s). The table below gives the conversion factors.
Additional Conversions for Graphic Design

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Kilobyte = 1000 bytes (sometimes 1024 bytes)</td>
<td></td>
</tr>
<tr>
<td>1 Megabyte = 1 million bytes</td>
<td></td>
</tr>
<tr>
<td>1 Gigabyte = 1000 megabytes</td>
<td></td>
</tr>
<tr>
<td>1 Terabyte = 1000 gigabytes</td>
<td></td>
</tr>
<tr>
<td>1 Petabyte = 1000 terabytes</td>
<td></td>
</tr>
<tr>
<td>1 Exabyte = 1000 petabytes</td>
<td></td>
</tr>
<tr>
<td>1 Zettabyte = 1000 exabytes</td>
<td></td>
</tr>
<tr>
<td>1 Yottabyte = 1000 zettabytes</td>
<td></td>
</tr>
</tbody>
</table>

In problems 125 – 129, use the table above to make the necessary conversions.

125. Convert 60 gigabytes to megabytes.
126. Convert 7,340,000 bytes to kilobytes.
127. Convert 520,000 gigabytes to terabytes.
128. Convert 1 yottabyte to terabytes.
129. Convert 5 gigabytes to exabytes.

FUN FACT: The **point** measurement system was created in 1886 by the U.S. Typefounders Association. This system has two units of measure: 1) point, and 2) pica. Points are used to measure the height of typefaces and vertical line spacing while picas are used to determine lengths of type lines and depths of type. A **line gauge** is a device used to measure point sizes and leading of printed type. A table of conversion factors for points and picas are given on the next page.

![Line Gauge](image)
### Additional Conversions for Graphic Design

#### Point Measurement Equivalents:

- 1 in. $\approx 72$ points
- 1 in. $\approx 6$ pica
- 1 point $\approx \frac{1}{72}$ in.
- 1 pica $= 12$ points

In problems 130 – 135, use the table above to make the necessary conversions.

130. The measurements of a newspaper ad is 4 inches by 3.75 inches. What are the measurements in pica?
131. A line of type is 42 pica long. What is the length in points?
132. A page of type measures 256 points wide. What is the width in inches?
133. A series of copy measurements includes 3 pica and 3 points; 12 pica; 12 pica and 3 point; and 12 pica and 6 points. What is the total measurement in pica and points?
134. A series of copy measurements includes 4 pica and 4 points; 11 pica; 13 pica and 5 points; and 12 pica and 6 points. What is the total amount of these measurements in pica and points?
135. A computer copy prep. artist keyboards two lines of type that are 27 pica 7 point long and 64 pica 5 point long. How much longer, in pica and points, is the second line than the first one?

### 7. APPLICATIONS FOR INTEGRATED ENERGY TECHNOLOGY

In problems 136 – 139, use the conversion factor 1 kilowatt (kW) = 1000 watts (W) to convert the following. Round decimal answers to the nearest hundredth.

136. The average home consumes 6,408,000 watts of energy per month. How many kW does the average home consume in a month?
137. A business consumes 7.6 kW in a week. How many watts does the business consume in a week?
138. A home consumes 7,520,000 watts of energy per month (30 days). How many kW per hour does this home consume?
139. A business consumes 114,000,000 watts of energy per year (365 days). How many kW per hour does this business consume?

**FUN FACT:** A *Kelvin (K)*, developed by Lord Kelvin in the mid 1800’s is an SI unit that is used to measure temperature. The equations on the next page are used to convert from Kelvin to Fahrenheit and Kelvin to Celsius.
Converting to Kelvin

\[
K = C + 273.15 \\
C = K - 273.15 \\
K = \frac{5}{9}(F - 32) + 273.15 \\
F = \frac{9}{5}(K - 273.15) + 32
\]

Note: \( F = \text{Fahrenheit}, \ C = \text{Celsius} \)

In problems 140 – 147, use the table above to convert each temperature.

140. Determine the Kelvin temperature for water boiling at 100 °C.
141. Determine the Kelvin temperature for water freezing at 0 °C.
142. Determine the Kelvin temperature for 32 °F.
143. Determine the Kelvin temperature for 77 °F.
144. Determine the degrees Celsius for 315 K.
145. Determine the degrees Celsius for -35 K.
146. Determine the degrees Fahrenheit for 295 K.
147. Determine the degrees Fahrenheit for 318.15 K.

8. APPLICATIONS FOR PROCESS TECHNOLOGY

FUN FACT: In chemical engineering the U.S. relies on the Rankine scale for measuring temperature when working in thermodynamic related disciplines such as combustion. Named after the Scottish physicist William John Macquorn Rankine who proposed it in 1859, the Rankine scale is an absolute scale based on the Fahrenheit increment.

Converting to and from Degrees Rankine (°R)

\[
R = F + 459.67 \quad \text{or} \quad F = R - 459.67 \\
R = \frac{9}{5}C + 491.67 \quad \text{or} \quad C = \frac{5}{9}(R - 491.67) \\
R = \frac{9}{5}K \quad \text{or} \quad K = \frac{5}{9}R
\]

Note: \( F = \text{Fahrenheit}, \ C = \text{Celsius}, \ K = \text{Kelvin} \)
In problems 148 – 153, use the table on the previous page to convert each temperature.

148. Determine the degrees Rankine for water boiling at 100 °C.
149. Determine the degrees Rankine for water freezing at 32 °F.
150. Determine the degrees Rankine for 105 K.
151. Determine the degrees Celsius for 315 °R.
152. Determine the degrees Fahrenheit for -35 °R.
153. Determine the temperature Kelvin for 413.5 °R.

The table on the following page will include units of energy, pressure, flow, and density, some of which we have not yet discussed. Let us first define those units that we haven’t yet defined.

**Units of Energy:**

**Joule (J):** This is the basic energy unit of the metric system. It is ultimately defined in terms of the meter, kilogram, and second.

**Calorie (cal):** Historically, one calorie is the amount of heat required to raise the temperature of 1 gram of water by 1 °C, from 14.5 °C to 15.5 °C. More recently the calorie has been defined in terms of the joule.

**Foot-Pound Force (ft-lb f):** This is the energy transferred on applying a force of one pound-force through a displacement of one foot.

**British Thermal Unit (Btu):** This is the English system version of the calorie.

**Kilowatt-hour (kW-h):** The kilowatt-hour is a standard unit of electricity production and consumption. Note that one kilowatt = 1000 watts.

**Therm:** In discussions of natural gas production and consumption it is common to use the unit therm.

**Units of Pressure:**

**Pounds Per Square Inch (PSI):** The pressure resulting from a force of one pound-force applied to an area of one square inch.

**Pounds Per Square Inch Gauge (PSIG):** A unit of pressure relative to the surrounding atmosphere.

**Pounds Per Square Inch Absolute (PSIA):** Measures pressure relative to a vacuum (such as that in space). This is the gauge pressure added to the local atmospheric pressure.

**Kilopascal (kPa):** Means 1000 Pascals, where a pascal (Pa) is the SI derived unit of pressure named after the French mathematician, physicist, inventor, writer, and philosopher Blaise Pascal.

**Standard Atmosphere (atm):** Since temperature and air pressure may vary from place to place it is necessary to define the standard atmospheric pressure for testing and documentation of chemical and physical processes. The standard atmospheric pressure is approximately $10^5$ Pascals.
Units of Density:

**Density** is defined to be mass divided by volume. The U.S. customary unit for mass is slug in which 1 slug = 14.59 kg. Therefore, the unit for density in the U.S. customary system is slug/ft³. In the metric system, the unit for density is kg/m³.

### Additional Conversions for Process Technology

#### Units of Energy:

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cal = 4.184 J</td>
<td></td>
</tr>
<tr>
<td>1 ft-lb f = 1.356 J</td>
<td></td>
</tr>
<tr>
<td>1 ft-lb f = 3.766x10⁻⁷ kW - h</td>
<td></td>
</tr>
<tr>
<td>1 J = 0.73756 ft-lb f</td>
<td></td>
</tr>
<tr>
<td>1 Btu = 251.998 cal</td>
<td></td>
</tr>
<tr>
<td>1 Btu = 1055 J</td>
<td></td>
</tr>
<tr>
<td>1 Btu = 777.7 ft-lb f</td>
<td></td>
</tr>
<tr>
<td>1 Btu = 2.928x10⁻⁴ kW - h</td>
<td></td>
</tr>
<tr>
<td>1 kW-h = 3.6x10⁶ J</td>
<td></td>
</tr>
<tr>
<td>1 kW-h = 3414 Btu</td>
<td></td>
</tr>
<tr>
<td>1 Exajoule (EJ) = 10¹⁸ J</td>
<td></td>
</tr>
<tr>
<td>1 quadrillion Btu (quad) = 10¹⁵ Btu</td>
<td></td>
</tr>
<tr>
<td>1 Terawatt-year (TW-yr) = 8.76x10¹² kW - h</td>
<td></td>
</tr>
<tr>
<td>1 Therm (therm) = 100,000 Btu</td>
<td></td>
</tr>
</tbody>
</table>

#### Units of Volume:

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ft³ = 7.48052 gal</td>
<td></td>
</tr>
<tr>
<td>1 gal = 0.003485 cubic meters (m³)</td>
<td></td>
</tr>
<tr>
<td>1 gal = 0.13346 ft³</td>
<td></td>
</tr>
<tr>
<td>1 gal = 231 cubic inches (in³)</td>
<td></td>
</tr>
</tbody>
</table>

#### Units of Flow:

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 gallon per minute (gpm)</td>
<td>2.228x10⁻³ ft³/s</td>
</tr>
<tr>
<td>1 ft³/s = 448.831 gpm</td>
<td></td>
</tr>
</tbody>
</table>

#### Units of Pressure:

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilopascal (kPa) = 0.145033 Pounds Per Square Inch (psi)</td>
<td></td>
</tr>
<tr>
<td>1 psi = 6.8948 kPa</td>
<td></td>
</tr>
<tr>
<td>1 Standard Atmosphere (atm) = 10⁵ Pascal (Pa)</td>
<td></td>
</tr>
<tr>
<td>x psig = (x + 14.7) psia</td>
<td></td>
</tr>
</tbody>
</table>

#### Units of Density:

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 slug per cubic foot (slug/ft³)</td>
<td>515.4 kilogram/cubic meter (kg/m³)</td>
</tr>
</tbody>
</table>
FUN FACT: In describing national or global energy budgets, it is common practice to use large-scale units based upon the joule, Btu, and kW-h, that is, we will use Exajoule (EJ), Quadrillion Btu (quad), Terawatt-year (TW-yr), and million British thermal unit (MBtu) to describe large scale energy measurements.

FUN FACT: The heat content of crude oil from different countries varies from about 5.6 million Btu (MBtu) per barrel to about 6.3 MBtu. The heat content of typical petroleum products varies even more. A nominal conversion factor is sometimes used for a barrel of crude oil, which is close to its actual average energy content: 1 barrel of oil equivalent = 5.80 MBtu.

In problems 154 – 182, use the table given on the previous page and dimensional analysis to answer the following questions. Round each decimal answer less than one to two non-zero digits (2 significant figures), otherwise round to the nearest hundredth.

154. Convert 60 kW-h to Joules.
155. Convert 14.5 kW-h to foot-pounds.
156. Convert 758 kW-h to British thermal units.
157. Convert 8,768,000 J to kilowatt-hours.
158. Convert 156,876 J to foot-pounds.
159. Convert 156.7 J to British thermal units.
160. Convert 7265 Btu to foot-pounds.
161. Convert 7.35 Btu to Joules.
162. Convert 25,382 Btu to kilowatt-hours.
163. Convert 560 cal to Joules.
164. Convert 4.32 TW-yr to kilowatt-hours.
165. Convert 78,000 Btu to therms.
166. Convert 678 ft³ to gallons.
167. Convert 42.8 gal to cubic feet.
168. Convert 4.5 gal to cubic inches.
169. Convert 62.7 in³ to gallons.
170. Convert 976.8 m³ to gallons.
171. Convert 78,540 gal to cubic meters.
172. Convert 72 gpm to cubic feet per second.
173. Convert 856.6 ft³/s to gallons per minute.
174. Convert 72,895 in³/hr to gallons per minute.
175. Convert 9024 kPa to pounds per square inch.
176. Convert 102,424 Pascals to pounds per square inch.
177. Convert 82 psi to kilopascals.
178. Convert the standard atmospheric pressure to pounds per square inch.
179. Convert 16 psig to psia.
180. Convert 52.7 psia to psig.
181. Convert 15.2 slugs/ft³ to kilograms per cubic meter.
182. Convert 978.7 kg/m³ to slugs per cubic foot.
9. APPLICATIONS FOR SKI AREA OPERATIONS

FUN FACT: Ski area managers estimate the time it takes to fill a pond, the number of hours it takes a front loader to dig a pond or create a jump, and the number of kilowatts of energy that will be used while completing these tasks. These computations ultimately help management determine the costs that the ski resort will incur over time. The following questions revolve around this theme.

In problems 183 – 186, use dimensional analysis to answer the following questions.

183. A front loader operator can dig one cubic yard in 15 seconds. What is the operators rate in cubic yards per hour?

184. A front loader operator digs 7 cubic yards per minute. How many cubic yards is this in an 8 hour working day?

185. A front loader operator digs 4,000 cubic feet of dirt in 9.6 hours. What is the operators rate in cubic yards per minute?

186. A front loader operator takes 125 minutes to dig 570 cubic feet of dirt. What is the operators rate in cubic yards per minute?

FUN FACT: In snowmaking, ski area management estimates the volume of snow and water and converts back and forth between the two. It is important to remember that 43,560 square feet is one acre, therefore, there are 43,560 cubic feet in an acre-foot of snow (i.e., we multiply 43,560 ft² by 1 ft to obtain the volume). This means that there is one acre of ground covered with one foot of snow. It takes on average 155,000 gallons of water to produce that acre foot of snow. Also, it has been determined that it takes about 1 kilowatt of power to put 300 gallons of water on a ski slope. The following questions revolve around this theme.

In problems 187 – 190, use dimensional analysis to answer the following questions. Round decimal answers to the nearest hundredth.

187. A ski resort needs to make 75 acre-feet of snow to put up on some of their trails. If it takes 155,000 gallons of water per acre-foot of snow, how many gallons of water is needed to complete this job?

188. A ski resort has 325,000 gallons of water to use to make snow. If 155,000 gallons of water produces one acre-foot of snow, how many acre-feet of snow can the ski resort produce?

189. It takes 1 kW of power to put 300 gallons of water on a ski slope and 155,000 gallons of water to produce one acre-foot of snow. a) How many kW of power will it take to produce 92 acre-feet of snow? b) If it costs $1.20 per kW of power, how much will it cost to make 92 acre-feet of snow?

190. It takes 1 kW of power to put 300 gallons of water on a ski slope and 155,000 gallons of water to produce one acre-foot of snow. a) How many kW of power will it take to produce 257.8 acre-feet of snow? b) If it costs $1.32 per kW of power, how much will it cost to make 257.8 acre-feet of snow?
Solutions to Module III:

1.  108 in.  2.  20.25 lb  3.  36 pt  4.  1440 s  5.  19.3 square yards  6.  1200 m  7.  9.5 m  8.  0.7 kg  9.  85 hg  10.  0.25 square meters  11.  3000 square centimeters  12.  0.045 L  13.  20 cc  14.  0.457 km  15.  10,500 mg  16.  500 micrograms  17.  45,000 g  18.  2500 mL  19.  80 c  20.  200 mL  21.  12,500 mg  22.  250 g  23.  0.5 g  24.  7.5 mg  25.  0.35 kg  26.  0.0005 L  27.  0.325 g  28.  0.5 dL  29.  27.5 dL  30.  8.5 cm  31.  300 gr  32.  40 fl dr  33.  6 pt  34.  32 fl oz  35.  4 dr  36.  2 oz  37.  3 fl oz  38.  15 gtt  39.  16 oz  40.  6 T  41.  12 t  42.  9 oz  43.  30 gtt  44.  3 T  45.  3 oz  46.  1.5 t  47.  1.5 c  48.  3 msg  49.  2.5 mL  50.  2.5 mL  51.  75 mL  52.  0.1 mL  53.  0.01 mL  54.  0.13 mL  55.  10 to 11 drops per minute  56.  400 units/hr  57.  9360 s  58.  40 pt  59.  80 c  60.  40.58 ch  61.  384 hr  62.  41.7 pounds  63.  Approximately 2.75 acres  64.  9.36 pints of gas, 0.28 pints of oil  65.  1.14 mph  66.  1:2,000 (in/ft)  67.  1:0.38 (in/mile)  68.  12,000 feet  69.  1 British Pound  70.  2 Euros  71.  15 Canadian Dollars  72.  780 Mexican Pesos  73.  $100.32  74.  532.83 Swiss Francs  75.  108.11 British Pounds  76.  285.86 Euros  77.  11,013.73 Mexican Pesos  78.  1960.13 Euros  79.  208 oz  80.  14 qt  81.  1.75 L  82.  0.25 square meters  83.  400 units/hr  84.  0.13 mL  85.  0.00000005 exabytes  86.  2500 g  87.  1.75 L  88.  0.5859 gal  89.  64.52 oz.  90.  1.6406 qt  91.  170.25 g  92.  1.3227 #  93.  21.28125 g  94.  2.5735 kg  95.  15.1923 L  96.  1.75 qt  97.  41 oz  98.  0.7968 gal  99.  1.25 #  100.  2.8125 oz  101.  39 tbsp  102.  4 servers  103.  800  104.  5460  105.  10,000  106.  10,000  107.  3375.19  108.  7029.1  109.  7994.492  110.  8400  111.  2,000,000  112.  Approximately 18° C  113.  Approximately 199° F  114.  0.004 inches thick  115.  0.768 inches.  116.  34 lbs.  117.  15 megapixels  118.  12.5 megapixels  119.  10,250,000 pixels  120.  15,050,000 pixels  121.  7.874 in  122.  3.543 in  123.  8.268 in  124.  11.811 in  125.  60,000 megabytes  126.  7167.97 kilobytes  127.  520 terabytes  128.  1,000,000,000,000 terabytes  129.  0.000000005 exabytes  130.  24 picas by 22.5 picas  131.  504 points  132.  3.56 in  133.  40 picas  134.  41 picas  135.  36 picas  136.  6408 kW  137.  7600 W  138.  10.44 kW/hr  139.  13.01 kW/hr  140.  373.15 K  141.  273.15 K  142.  273.15 K  143.  298.15 K  144.  41.85 °C  145.  -308.15 °C  146.  71.33 °F  147.  113 °F  148.  671.67 °R  149.  491.67 °R  150.  189 °R  151.  -98.15 °C  152.  -494.67 °F  153.  229.72 K  154.  216,000,000 J  155.  38,502,389.8 ft-lb f  156.  2,587,812 Btu  157.  2.44 kW-h  158.  115,705,46 ft-lb f  159.  0.15 Btu  160.  5,649,990.5 ft-lb f  161.  7754.25 J  162.  7.43 kW-h  163.  2343.04 J  164.  37,843,200,000,000 kW-hr  165.  7.8 therms  166.  5071.79 gal  167.  5.71 ft³  168.  1039.5 in³  169.  0.27 gal  170.  280,286.94 gal  171.  273.71 gal  172.  0.16 ft³/s  173.  385,104.15 gpm  174.  5.26 gpm  175.  1308.78 psi  176.  14.85 psi  177.  565.39 kPa  178.  14.5 psi  179.  30.7 psia  180.  38 psig  181.  7818.88 kg/m³  182.  1.90 slug/ft³  183.  240 yd³/hr  184.  3360 yd³  185.  6.94 yd³/min  186.  0.17 yd³/min  187.  11,625,000 gal  188.  2.10 acre-ft  189. a) 47,533.33 kW of power  b) $57,040  190. a) 133,196.67 kW of power  b) $175,819.60
Module IV
Polynomials: Operations

In applied mathematics, mathematicians constantly strive to create formulas that model real-life behavior. If the model produces reliable results, then it may be used as a formula. Mathematical models help scientists make predictions and test conjectures without necessarily having to conduct real experiments. All fields of study use mathematical models. For example, an EMT or nurse may need to use the formula for calculating body surface area to help administer medications for pediatric or fragile adult patients; a firefighter will use the formula for calculating the desired nozzle pressure for a hose; an accountant will use the formula for calculating compound interest; a professional chef will use the formula for determining the monthly cost of food sold; a graphic designer will use the formula for calculating ink coverage; a photographer will use the formula for calculating the bellows factor, which is needed to compute the f-stop adjustment; a solar PV system installer will need to use Ohm’s Law to calculate voltage; oil industry technicians may need to use Boyle’s law to describe the relationship between the absolute pressure and the volume of a gas as well as many other geometric formulas for the design of equipment; and ski area managers will need to use geometric formulas to help calculate volume for snowmaking. We will be defining and using each of these formulas and more in Module VI. In this module we will focus on all of the expressions and the mathematical operations with them that make the formulas come to life.

I. Expressions and Equations


It is often easy for one to confuse an equation with an expression. An equation “connects” expressions together with an equal sign. 2x + 3 = 7 is an example of an equation. An expression is a collection of constants and variables that are connected with a combination of arithmetic operations, such as +, −, ×, or ÷. Essentially, an expression “looks” like an equation without the “=” sign. 2x + 3 is an example of an expression. An algebraic expression contains at least one variable, while a numeric expression contains no variables (only constants, i.e. “numbers”).

In this module we will be: (1) Evaluating algebraic expressions given value(s) for the variable(s), (2) Simplifying algebraic expressions, that is, rewriting an algebraic expression in its simplest form, (3) Adding and subtracting polynomials, (4) Reviewing laws of exponents, (5) Multiplying and dividing polynomials, and (6) Creating expressions to help solve applications. We will begin with evaluating expressions.
Evaluating Expressions:
The direction “evaluate” means “find the value”.

1. Substitute the given variable(s) into the expression. Make sure to use parentheses to help avoid making careless mistakes.
2. Simplify the numeric expression by using the proper order of operations.

Example 1: Evaluate \(-x^2 - 5x\) when \(x = -3\).

Solution to Example 1:
First, we will substitute \(x = -3\) into the algebraic expression \(-x^2 - 5x\) to get \((-3)^2 - 5(-3)\).
Next, we will simplify the numeric expression using the proper order of operations.
\((-3)^2 - 5(-3) = -9 + 15 = 6\). Answer: 6

Example 2: Evaluate \(\frac{-6uv + 14}{3u - v^2}\), when \(u = -1\) and \(v = 4\).

Solution to Example 2:
First, we will substitute \(u = -1\) and \(v = 4\) into the algebraic expression \(-6uv + 14\) to get \(-6(-1)(4) + 14\).
Next, we will simplify the numeric expression using the proper order of operations.
\(-6(-1)(4) + 14 = \frac{24 + 14}{3(-1) - 16} = \frac{38}{-19} = -2\). Answer: -2

II. Polynomials

There are many different types of algebraic expressions. In this section we will learn the language used to identify a specific type of expression called a polynomial and we will begin the simplification process by combining like terms.

A monomial is an expression that consists of the product of constants and/or variables each raised to a power that is a whole number. \(3x^2\) is an example of a monomial; however, \(3x^{-2}\) is not a monomial since the exponent is not a whole number. A monomial is also known as a term. A coefficient is the numerical factor in a monomial. The coefficient of \(3x^2\) is 3. Two or more terms are considered “like terms” if they all have the same variables each raised to the
same exponents. For example, $3x^2y^3z$ and $-2x^2y^3z$ are like terms. Basically, the only thing that can be different about like terms are the coefficients. A **polynomial** is any monomial or sum of monomials. $2x^3 - xy + 7z - 8$ is an example of a polynomial. A **binomial** is a polynomial with exactly two terms. $5x + 3$ is an example of a binomial. A **trinomial** is a polynomial with exactly three terms. $x^2 - 3x + 2$ is an example of a trinomial. The **degree of a term (or monomial)** is the sum of the exponents of the variables that make up the monomial. For example, the degree of the monomial $7x^2y$ is 3 since the exponent on the variable $x$ is 2 and the exponent on the variable $y$ is 1, so the sum of 2 and 1 is 3. The **degree of a polynomial** is the largest of the degrees of the terms that make up the polynomial (after like terms have been combined). The degree of the polynomial $2x^3 - xy + 7z - 8$ is 3 since if we compare the degrees of each term ($2x^3$ has degree 3, $-xy$ has degree 2, $7z$ has degree 1, and $-8$ has degree 0) we see that degree 3 is the highest of all the terms that make up the polynomial. A polynomial is written in **descending order** if the terms are written from highest degree to lowest degree from left to right.

**Combining Like Terms:**

1. Look for terms that are like terms, that is, terms that have the same variables each raised to the same exponents.
2. Add or subtract just the coefficients of each like term. Leave the variables and exponents alone.
3. Write your answer in descending order (highest degree to lowest degree).

**Example 3:** Simplify the following expression and write your answer in descending order.

$$-3y^2 + 15y - 13 - 6y^2 + 9y^4 - 8y^3 - 12y^4 + 13 + 3y - 10y^4$$

**Solution to Example 3:**

It is helpful to identify the like terms. Striking through, circling or boxing like terms may help.

Since we need to simplify and write in descending order, we will start combining like terms with the highest degree, and then proceed in a descending fashion. First, we combine $9y^4$, $-12y^4$ and $-10y^4$ to get $-13y^4$. Then, since there is no like term with $-8y^3$, we will leave it alone. Next, we combine $-3y^2$ with $-6y^2$ to get $-9y^2$. Then we combine $15y$ with $3y$ to get $18y$. Finally, we add $-13$ with $13$ to get $0$. Our answer is $-13y^4 - 8y^3 - 9y^2 + 18y$.

**Answer:** $-13y^4 - 8y^3 - 9y^2 + 18y$
III. Adding and Subtracting Polynomials

Khan Academy Resources:


Adding or Subtracting Polynomials:
1. When adding polynomials, simply remove any parentheses and then combine like terms.
2. When subtracting polynomials, you must use the distributive property on the expression with the “−” in front to remove parentheses. This means that all of the signs of the expression that you are subtracting will change. After parentheses are removed you then add like terms.

Example 4: Add and write your answer in descending order. \((x^2 - 9x + 2) + (-x^2 + 2x - 8)\)

Solution to Example 4:
Since the problem is addition we may remove the parentheses to get \(x^2 - 9x + 2 - x^2 + 2x - 8\). We will now combine like terms and write in descending order to get \(-7x - 6\). Answer: \(-7x - 6\)

Example 5: Subtract and write your answer in descending order. \((2x^4 + 3x) - (5x^3 + 4x + 3)\)

Solution to Example 5:
Since the problem is subtraction we must use the distributive property with the “−“(thinking of it as “-1”) to remove the parentheses. We get \(2x^4 + 3x - 5x^3 - 4x - 3\). We will now combine like terms and write in descending order to get \(2x^4 - 5x^3 - x - 3\).
Answer: \(2x^4 - 5x^3 - x - 3\)
IV. Laws of Integer Exponents

To be able to multiply and divide polynomials we must first understand the laws governing integer exponents. In this section we will introduce the laws of exponents, then we will apply them to problems involving scientific notation. After that we will apply them to multiplication and division of polynomials.

Laws of Exponents (assume \( m, n > 0 \)):

1. \( a^1 = a \)
2. \( a^0 = 1, \ a \neq 0 \)
3. \( 0^0 = \text{Undefined} \)
4. \( a^m \cdot a^n = a^{m+n} \)
5. \( \frac{a^m}{a^n} = a^{m-n}, \ a \neq 0 \)
6. \( (a^m)^n = a^{mn} \)
7. \( (ab)^n = a^n b^n \)
8. \( \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n}, \ b \neq 0 \)

Laws of Negative Exponents (assume \( n > 0 \)):

1. \( a^{-n} = \frac{1}{a^n}, \ a \neq 0 \)
2. \( \frac{1}{a^{-n}} = a^n, \ a \neq 0 \)
3. \( \left( \frac{a}{b} \right)^{-n} = \left( \frac{b}{a} \right)^n, \ a, b \neq 0 \)

WARNING: DON’T FALL INTO THIS TRAP! \( (a + b)^n \neq a^n + b^n \)

For example \( (2 + 3)^4 \neq 2^4 + 3^4 \), that is, \( 625 \neq 97 \)

Example 6: Use the laws of exponents to simplify the following and write your answer without negative exponents.

a) \( x^3 \cdot x^{-7} \cdot x^2 \cdot x^0 \)
\[ b) \quad \frac{x^2 x^4}{x^{-4}} \]
\[ c) \quad \left( \frac{y^5 y^{-2}}{y \cdot y^{-3}} \right)^2 \]

Solutions to Example 6:

a) Using rule number 4 under “laws of exponents”, since we have a common base, we can simply add the exponents giving us \( x^3 \cdot x^{-7} \cdot x^2 \cdot x^0 = x^{3+(-7)+2+0} = x^{-2} \). To rewrite \( x^{-2} \) we must use rule number 1 under “negative exponents” to get \( x^{-2} = \frac{1}{x^2} \).

Answer: \( \frac{1}{x^2} \)

b) Using rule number 4 under “laws of exponents”, since we have a common base, we can simply add the exponents in the numerator giving us \( \frac{x^{2+4}}{x^{-4}} = \frac{x^6}{x^{-4}} \). Using rule number 5 under “laws of exponents” we can subtract the exponents to get \( \frac{x^6}{x^{-4}} = x^{6-(-4)} = x^{10} \).

Answer: \( x^{10} \)

c) Using a combination of rules 7 and 8 under “laws of exponents” we can multiply the exponent outside the parentheses to each exponent inside the parentheses giving us \( \left( \frac{y^5 y^{-2}}{y \cdot y^{-3}} \right)^2 = \frac{y^{10} y^{-4}}{y^2 y^{-3}} \). Now, using rule number 4 under “laws of exponents” we can add the exponents in the numerator and add the exponents in the denominator to get \( \frac{y^{10} y^{-4}}{y^2 y^{-3}} = \frac{y^{10+(-4)}}{y^{2+(-3)}} = \frac{y^6}{y^{-1}} \). Using rule number 5 under “laws of exponents” we can subtract the exponents giving us \( \frac{y^6}{y^{-1}} = y^{6-(-1)} = y^7 \).

Answer: \( y^7 \)
V. Laws of Exponents with Scientific Notation

Khan Academy Resources:
https://www.khanacademy.org/math/pre-algebra/pre-algebra-exponents-radicals/pre-algebracomputing-scientific-notation/v/scientific-notation-example-2


Example 7: For the following, perform the operation and write your answer in scientific notation.

a) \((3.7 \times 10^{-4})(4.8 \times 10^{-3})\)

b) \((2.4 \times 10^{-8})(6.5 \times 10^{32})\)

Solutions to Example 7:

a) In this problem, since it is all multiplication, and multiplication is commutative, we can rewrite the product as follows: \((3.7 \times 10^{-4})(4.8 \times 10^{-3}) = (3.7 \cdot 4.8) \times (10^{-4} \cdot 10^{-3})\). Now, after multiplying and using the laws of exponents, we get: 
\((3.7 \cdot 4.8) \times (10^{-4} \cdot 10^{-3}) = 17.76 \times 10^{-7}\). This answer is not quite in scientific notation since 17.76 is not a number between 1 and 10. To write it in scientific notation, we must do the following: 
\(17.76 \times 10^{-7} = 1.776 \times 10^{1} \times 10^{-7} = 1.776 \times 10^{-6}\).

b) \((2.4 \times 10^{-8})(6.5 \times 10^{32}) = \left(\frac{2.4 \cdot 6.5}{5.2}\right) \times \left(\frac{10^{-8} \cdot 10^{32}}{10^{18}}\right) = 3 \times 10^{6}\). Answer: \(3 \times 10^{6}\)

VI. Multiplying Polynomials

Khan Academy Resources:


When multiplying polynomials, we need to first understand the distributive property.

**Distributive Property:** For any numbers \( a, b, \) and \( c, \) \( a(b + c) = ab + ac. \)

This means that the product of a number and a sum can be written as the sum of two products. For instance, suppose we were multiplying \( 24 \times 12. \) We could rewrite this product in a way that makes multiplying these numbers easier, such as:

\[
\begin{align*}
24 \times 12 &= 24(10 + 2) \\
&= (24 \times 10) + (24 \times 2) \\
&= 240 + 48 \\
&= 288,
\end{align*}
\]

\[\text{OR}\]

\[
\begin{align*}
24 \times 12 &= 12(12 + 12) \\
&= (12 \times 12) + (12 \times 12) \\
&= 144 + 144 \\
&= 288.
\end{align*}
\]

We use the distributive property extensively when multiplying polynomials.

**Multiplying Polynomials:**

When multiplying two polynomials
- 1. Use the distributive property repeatedly until you have distributed each of the terms of one of the polynomials to each of the terms of the other polynomial.
- 2. Use the laws of exponents to multiply monomials.
- 3. Combine like terms. If you are multiplying three or more polynomials, multiply two together, then multiply that product to the remaining polynomial, etc. It helps to look for patterns when multiplying polynomials.

Note: You can determine the number of terms in the product (before combining like terms) by noting the number of terms in the polynomials. If one polynomial has \( m \) terms, and the other has \( n \) terms, then their product (before simplifying) will have \( m \times n \) terms.
Example 8: Multiply the following and simplify.

a) \( xy^2 \left( x^2 + 4y \right) \)

b) \((3x - 4)(x - 5)\)

c) \((x - 3y)(x^2 + 3xy + 9y^2)\)

d) \((2x - 1)^2\)

e) \((4x + 9)(4x - 9)\)

Solutions to Example 8:

a) Using the distributive property we get: \( xy^2 \left( x^2 + 4y \right) = (xy^2)(x^2) + (xy^2)(4y) \). Using the laws of exponents we get: \((xy^2)(x^2) + (xy^2)(4y) = x^3y^2 + 4xy^3\). Since there are no like terms, this is our answer. **Answer:** \( x^3y^2 + 4xy^3\)

b) Using the distributive property twice (first with \(3x\), then with \(-4\)) we get: \((3x - 4)(x - 5) = (3x)(x) + (3x)(-5) + (-4)(x) + (-4)(-5)\). Using the laws of exponents we get: \((3x)(x) + (3x)(-5) + (-4)(x) + (-4)(-5) = 3x^2 - 15x - 4x + 20\). Combining like terms we get: \(3x^2 - 15x - 4x + 20 = 3x^2 - 19x + 20\). **Answer:** \(3x^2 - 19x + 20\)

c) \((x - 3y)(x^2 + 3xy + 9y^2)\)

\[\begin{align*}
&= (x)(x^2) + (x)(3xy) + (x)(9y^2) + (-3y)(x^2) + (-3y)(3xy) + (-3y)(9y^2) \\
&= x^3 + 3x^2y + 9xy^2 - 3x^2y - 9xy^2 - 27y^3 \\
&= x^3 - 27y^3
\end{align*}\]

**Answer:** \(x^3 - 27y^3\)

d) \((2x - 1)^2 = (2x - 1)(2x - 1)\)

\[\begin{align*}
&= (2x)(2x) + (2x)(-1) + (-1)(2x) + (-1)(-1) \\
&= 4x^2 - 2x - 2x + 1 \\
&= 4x^2 - 4x + 1
\end{align*}\]

**Answer:** \(4x^2 - 4x + 1\)

e) \((4x + 9)(4x - 9)\)

\[\begin{align*}
&= (4x)(4x) + (4x)(-9) + (9)(4x) + (9)(-9) \\
&= 16x^2 - 36x + 36x - 81 \\
&= 16x^2 - 81
\end{align*}\]

**Answer:** \(16x^2 - 81\)
VII. Special Products

Khan Academy Resources:


In terms of multiplying polynomials, *special products* are those polynomials that multiply to give you either a *perfect square trinomial*; *difference of squares*; *difference of cubes*; or *sum of cubes*. Recognizing special products helps us tremendously when factoring. We will be factoring in the next module, so play close attention *NOW* to the patterns of special products.

**Perfect Square Trinomials:**

\[(A + B)^2 = A^2 + 2AB + B^2\]

\[(A - B)^2 = A^2 - 2AB + B^2\]

**Example 9:** Multiply the following and simplify.

a) \((x + 1)^2\)

b) \((2x - 5)^2\)

**Solutions to Example 9:**

a) \((x + 1)^2 = (x + 1)(x + 1)\)

\[= (x)(x) + (x)(1) + (1)(x) + (1)(1)\]

\[= x^2 + x + x + 1\]

\[= x^2 + 2x + 1\]

**Answer:** \(x^2 + 2x + 1\)

This result is known as a “Perfect Square Trinomial” since \((x + 1)^2 = (x)^2 + 2(x)(1) + (1)^2 = x^2 + 2x + 1\)

b) \((2x - 5)^2 = (2x - 5)(2x - 5)\)

\[= (2x)(2x) + (2x)(-5) + (-5)(2x) + (-5)(-5)\]

\[= 4x^2 - 10x - 10x + 25\]

\[= 4x^2 - 20x + 25\]

**Answer:** \(4x^2 - 20x + 25\)
This result is known as a “Perfect Square Trinomial” since
\[(2x - 5)^2 = (2x)^2 - 2(2x)(5) + (5)^2 = 4x^2 - 20x + 25\]

Notice that with perfect square trinomials the **first term** is a perfect square, the **last term** is a perfect square, and the **middle term** is the product of the square-roots of the first and last terms, doubled. The sign of the middle term is determined by the sign in the binomial that was being squared.

**Difference of Squares:**
\[(A + B)(A - B) = A^2 - B^2\]

**Example 10:** Multiply the following and simplify.

a) \((x + 2)(x - 2)\)
b) \((3x - 7)(3x + 7)\)

Solutions to Example 10:

a) 
\[(x + 2)(x - 2)\]
\[= (x)(x) + (x)(-2) + (2)(x) + (2)(-2)\]
\[= x^2 - 2x + 2x - 4\]
\[= x^2 - 4\]
**Answer:** \(x^2 - 4\)

This result is known as a “Difference of Squares” since
\[(x + 2)(x - 2) = (x)^2 + 2x - 2x - (2)^2 = (x)^2 - (2)^2 = x^2 - 4\]

b) 
\[(3x - 7)(3x + 7)\]
\[= (3x)(3x) + (3x)(7) + (-7)(3x) + (-7)(7)\]
\[= 9x^2 + 21x - 21x - 49\]
\[= 9x^2 - 49\]
**Answer:** \(9x^2 - 49\)

This result is known as a “Difference of Squares” since
\[(3x - 7)(3x + 7) = (3x)^2 + 21x - 21x - (7)^2 = (3x)^2 - (7)^2 = 9x^2 - 49\]

Notice that with difference of squares the first term is a perfect square, the last term is a perfect square, and the operation connecting the terms is subtraction, hence **difference of squares.**
**Difference of Cubes:**
\[(A - B)(A^2 + AB + B^2) = A^3 - B^3\]

**Sum of Cubes:**
\[(A + B)(A^2 - AB + B^2) = A^3 + B^3\]

**Example 11:** Multiply the following and simplify.

a) \((x - 2)(x^2 + 2x + 4)\)

\[
= (x)(x^2) + (x)(2x) + (x)(4) + (-2)(x^2) + (-2)(2x) + (-2)(4)
= x^3 + 2x^2 + 4x - 2x^2 - 4x - 8
= x^3 - 8
\]

**Answer:** \(x^3 - 8\)

This result is known as a “Difference of Cubes” since \((x - 2)(x^2 + 2x + 4) = (x)^3 - (2)^3 = x^3 - 8\)

b) \((2x + 3)(4x^2 - 6x + 9)\)

\[
= (2x)(4x^2) + (2x)(-6x) + (2x)(9) + (3)(4x^2) + (3)(-6x) + (3)(9)
= 8x^3 - 12x^2 + 18x + 12x^2 - 18x + 27
= 8x^3 + 27
\]

**Answer:** \(8x^3 + 27\)

This result is known as a “Sum of Cubes” since \((2x + 3)(4x^2 - 6x + 9) = (2x)^3 + (3)^3 = 8x^3 + 27\)

Notice that a difference of cubes is two perfect cubes being subtracted, and a sum of cubes is two perfect cubes being added.

**Example 12:** In example 8, from parts a – d, there were two special products. Name them.

Solutions to Example 12:

a) Example 8, part a was **not a special product**.

b) Example 8, part b was **not a special product**.

c) Example 8, part c is a **Difference of Cubes**.

d) Example 8, part d is a **Perfect Square Trinomial**.

e) Example 8, part e is a **Difference of Squares**.
VIII. Dividing Polynomials

Khan Academy Resources:


Dividing Monomials by Monomials:
1. Divide numerical values by looking for common factors.
2. Use the laws of exponents to reduce variable common factors.

Dividing Polynomials by Monomials:

When dividing Polynomials by Monomials, use the rule: \( \frac{a + b}{c} = \frac{a}{c} + \frac{b}{c} \).

Example 13: Divide the following. \( \frac{-32x^4y^5z^8}{4x^2yz^3} \)

Solution to Example 13:
First we will divide the numerical values. Since 4 divides evenly into -32 we get:

\[ \frac{-32x^4y^5z^8}{4x^2yz^3} = \frac{-8x^4y^5z^8}{x^2yz^3} \]

Now use the laws of exponents to reduce variable common factors:

\[ \frac{-8x^4y^5z^8}{x^2yz^3} = -8x^{(4-2)}y^{(5-1)}z^{(8-3)} = -8x^2y^4z^5 \]

Answer: \( -8x^2y^4z^5 \)

Example 14: Divide the following. \( \frac{48x^4y^2 + 36x^5y^3 - 6x^2y}{6x^2y} \)

Solution to Example 14:

\[ \frac{48x^4y^2 + 36x^5y^3 - 6x^2y}{6x^2y} = \frac{48x^4y^2}{6x^2y} + \frac{36x^5y^3}{6x^2y} - \frac{6x^2y}{6x^2y} = 8x^2y + 6x^3y^2 - 1 \]

Answer: \( 8x^2y + 6x^3y^2 - 1 \)
Simplifying Expressions with Multiple Operations:
Use the order of operations:
1. Simplify what’s inside Parentheses ( ), Brackets [ ], Braces { }, Absolute Value | |
   Square Roots \( \sqrt{ } \), and a division bar
2. Simplify Expressions with Exponents
3. Multiplication and Division (reading from left to right)
4. Addition and Subtraction (reading from left to right)

Example 15: Simplify the following. \( 5(2x - 7) - [4(2x - 3) + 2] \)

Solution to Example 15:
\[
5(2x - 7) - [4(2x - 3) + 2] = 5(2x - 7) - [8x - 12 + 2] \\
= 5(2x - 7) - [8x - 10] = 10x - 35 - [8x - 10] \\
= 10x - 35 - 8x + 10 \\
= 2x - 25 \\
Answer: 2x - 25

IX. Polynomials Used In Mathematical Modeling

In all of the examples given up to this point, expressions were provided for us to evaluate and/or simplify. In mathematical modeling, we will not be given the expressions or equations, rather, we will be asked to develop them on our own. Once we have developed a mathematical model, we will use it to make predictions. This section will help us develop the skills needed to translate a real-life application to an algebraic expression.

Suppose we want to evaluate the net income for running a wedding photography business. In order for us to do this we’ll need to determine how the business will earn income, that is, determine its total revenue, as well as the total costs associated with running the business. Total costs include fixed costs (costs that never change) and variable costs (costs that change according to the frequency or rate). There are many variables that determine revenue and cost expressions. For instance, a wedding photographer may earn revenue by charging an hourly rate or a sitting fee, a fee to travel distances, and a fee for each different wedding photo package that the photographer has to offer its customers. Costs to run the same business may include the cost of equipment, materials, travel, production of each wedding photo package, advertisement, and facility rental (including lease, insurance, utilities, etc.). The table below outlines some possible sources of revenue and costs in a month for a wedding photography business.
Wedding Photography Revenue

Sitting Fee: $50 Per Customer
Travel Reimbursement: $.50 Per Mile
$200 for package 1
$250 for package 2
$300 for package 3
$12 for each additional individual photo

Wedding Photography Costs

Equipment: $750 per month fixed
Maintenance: $20 per month fixed
Materials:
$25 for package 1
$35 for package 2
$50 for package 3
$2 for each individual photo
Travel: $.20 per mile
Advertisement: $35 per month fixed
Facility: $1250 per month fixed

Since all of the sources of revenue in the above table are not fixed, we need to use a variable to define each unknown: Let c = number of customers; m = number of miles driven; x = number of package 1 sold; y = number of package 2 sold; z = number of package 3 sold; p = number of each additional photo sold. The same variables can be used to define the variable costs. The fixed costs we can simply add. We can now write expressions representing sources of revenue and cost. The following table lists the expressions that we will use to determine revenue and cost.

<table>
<thead>
<tr>
<th>Wedding Photography Revenue (in $)</th>
<th>Wedding Photography Costs (in $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sitting Fee: 50c</td>
<td>Travel: 0.2m</td>
</tr>
<tr>
<td>Travel Reimbursement: 0.5m</td>
<td>Package 1: 25x</td>
</tr>
<tr>
<td>Package 1: 200x</td>
<td>Package 2: 35y</td>
</tr>
<tr>
<td>Package 2: 250y</td>
<td>Package 3: 50z</td>
</tr>
<tr>
<td>Package 3: 300z</td>
<td>Additional individual photo: 2p</td>
</tr>
<tr>
<td>Additional individual photos: 12p</td>
<td>Advertisement: 35 per month fixed</td>
</tr>
<tr>
<td></td>
<td>Facility: 1250 per month fixed</td>
</tr>
<tr>
<td></td>
<td>Equipment: 750 per month fixed</td>
</tr>
<tr>
<td></td>
<td>Maintenance: 20 per month fixed</td>
</tr>
</tbody>
</table>

We can combine all of the sources of revenue to write an expression for the total revenue. The total monthly revenue for running this business will be: 50c + 0.5m + 200x + 250y + 300z + 12p. We can do the same for the total costs. The total monthly cost for running this business is: 0.2m + 25x + 35y + 50z + 2p + 2055. Finally, since net = revenue – cost, we can write an expression for the net:
Net = 50c + 0.5m + 200x + 250y + 300z + 12p – (0.2m + 25x + 35y + 50z + 2p + 2055)

Since we have like terms, the formula can be simplified to:

Net = 50c + 0.3m + 175x + 215y + 250z + 10p – 2055

Next, suppose that in a month’s time the business has 11 customers, a total of 750 miles were driven to meet customers, a total of 5 of package one were sold, a total of 2 of package two were sold, a total of 4 of package three were sold, and 15 additional photos were sold. How much did the photography business net during that month? Is it a profit or a loss? To answer this question we will evaluate our net expression for each of the given values.

Net = 50(11) + 0.3(750) + 175(5) + 215(2) + 250(4) + 10(15) – 2055
Net = 550 + 225 + 875 + 430 + 1000 + 150 – 2055
Net = $1175

Under the conditions that were given, we can see that the photography business earned a net of $1175, which is a profit.

**Homework Set:**

In problems 1 – 5, evaluate each expression using the given values.

1. \(5y – 7\), when \(y = -2\)
2. \(3y – 5(y + 2)\), when \(y = 4\)
3. \(|x^2 + y|\), when \(x = 2\) and \(y = -10\)
4. \(\frac{a^3 - 4a}{a(a - 3)}\), when \(a = -2\)
5. \(4x^2 + 5x - 5\) for \(x = -9\).

In problems 6 – 7, simplify and write the polynomial in descending order.

6. \(13b^3 - 9b^2 + 5b - 4b^2 + 22b^3\)
7. \(3x^2 - 2x + 3 - 5x^2 - 1 - x\)

In problems 8 – 11, add the following expressions.

8. \((7a + 3b) + (-4a + 8b) + (-6a - b)\)
9. \((2x^2 - 3xy) + (9y^2 - 2xy) + (x^2 - yx)\)
10. \((0.2ab^2c - 5.1ab) + (-1.2a^2bc + 3.7ab^2c) + (5.1ab - 3.9a^2bc)\)
11. \((\frac{1}{2}a - \frac{2}{3}b + \frac{1}{4}c) + \left(-\frac{3}{5}a + \frac{1}{4}b - 2c\right) + \left(-\frac{2}{3}a + 4b - \frac{1}{2}c\right)\)

In problems 12 – 15, subtract the following expressions.

12. \((-a + 3b) - (3a - 5b)\)
13. \((9x - 6xy) - (-2xy - 5x)\)
14. Subtract \(-5x^2 + 7xy\) from \(3y^2 + 4xy + 2x^2\)
15. Subtract \(9a + 8b + 6c\) from \(21a + 18b + 3c\)

In problems 16 – 24, simplify.

16. \((5mn + 4nt) + (-3nt - 2mn) - (16mn + nt)\)
17. \((x^5 + 5x^4 - 9x^2 + 11) + (-2x^5 - 14x^4 - 6x^3 + 5)\)
18. \((t^5 - 9t^4 - 11t^3 + t + 15) - (-t^5 + 4t^4 - 11t^3 + 6t^2 + 20)\)
19. \((9xy + 8y^2) - (6x^2 - 8xy) + (5x^2 - 3xy + 7y^2)\)
20. \(-[7a + 5b] - 5[2a - 3b]\)
21. \(5[-3(6a - 2\{5a - 4 + 6b\}) - 3a] + a\)
22. \(-[4a(6a - 5b) + 5(6a^2 - 6b) - 6a(3 - 5a)]\)
23. \(-[3(5a(11a - 5b)) + 6] - 5[4(a(4a + b))]\)
24. \(a - \{2a - 1 + \{a + 2(3a - 4)\}\}\)

25. For the rectangle to the right,

a) Write an expression for the perimeter.

b) Write an expression for the area.

c) Evaluate your answer in b) when \(x = 5\).

In problems 26 – 28, answer true or false. If the answer is false, give an example or explanation why it is false.

26. \(3^{-7} = \frac{1}{-3^7}\)
27. \(\frac{1}{y^4} = y^{-4}\)
28. \(0^0 = 0\)

In problems 29 – 31, multiply or divide and write your answer in scientific notation.

29. \(\frac{8.1 \times 10^{-4}}{1.8 \times 10^3}\)
30. \((3.2 \times 10^{-2})(4.7 \times 10^{-4})\)
31. A strand of DNA is about 1.5 m long and $1.3 \times 10^{-10}$ cm wide. How many times longer is DNA than it is wide?

In problems 32 – 51, use the laws of exponents to simplify. Write your answer in positive exponent form. Assume all variables represent positive numbers.

32. $x^3 \cdot x^2 \cdot x^4$
33. $(x + 4)^3(x + 4)^3$
34. $\frac{3^8}{3^2}$
35. $\frac{(5x)^7}{(5x)^4}$
36. $(-3y^2)^3$
37. $(2x^3)^3(-5x^5)^2$
38. $6(x^7)^2(-2x^5)^5$
39. $t^{-2} \cdot t^{-5}$
40. $\left(\frac{x^3 y^4}{x^7 y^{-2}}\right)^0$
41. $(3a^{-1}b^3)^{-2}$
42. $\left(\frac{a^2b}{c}\right)^3$
43. $(y^{-5x})(y^{-2x})$
44. $\frac{(-5x^7)^6}{(-5x^3)^4}$
45. $\frac{(-3x^3)^5}{2y^5}$
46. $[3x(x+1)]^4$
47. $x^{3k} \cdot x^4$
48. $(2x^4)^2(3x^{-1})^2$
49. $\left(\frac{ac^{-2}}{d^2b^4}\right)^{-4}$
50. $\left(\frac{2ab^4}{3b^5}\right)^{-3}$
In problems 52 – 59, multiply and simplify the following.

52. \((x + 3)(x + 3)\)
53. \((5x - 2y)^2\)
54. \((a + b)(a - b)\)
55. \((2y - 1)(2y + 1)\)
56. \((2x - 3y)(2x + 3y)\)
57. \(x^2y\left(5x^2 + y^2\right)(2x^2 - 3y^2)\)
58. \((3x - 5)(9x^2 + 15x + 25)\)
59. \((15a + 11)(3a - 13)\)

In problems 60 – 61, divide the following.

60. \(-\frac{63x^5y^4z^8a}{-7x^4y^2z^5a}\)
61. Divide \(9x^2y^2 + 3x^2y - 6xy^2\) by \(-3xy\).

1. APPLICATIONS FOR EMT/MEDICAL ASSISTANT/NURSING

In problems 62 – 65, evaluate the expression used for calculating body surface area: \(\sqrt{\frac{h \cdot w}{3600}}\) for each given values \(h\) and \(w\). Round each answer to the nearest hundredth.

62. \(h = 160, w = 57.8\)
63. \(h = 175.32, w = 72\)
64. \(h = 180 \frac{1}{2}, w = 92.34\)
65. \(h = 184.92, w = 98 \frac{3}{4}\)

2. APPLICATIONS FOR FIRE SCIENCE

In problems 66 – 67, evaluate the expression used for calculating force: \(m \cdot a\) for each given values \(m\) and \(a\). Round each answer to the nearest hundredth.

66. \(m = 5.34, a = -32\)
67. \(m = 2.875, a = -10\)
In problems 68 – 69, evaluate the expression used for calculating the projected time for a fire to spread a certain distance: \( \frac{s}{r} \) for each given values \( s \) and \( r \). Round each answer to the nearest hundredth.

68. \( s = 13.34, r = 2.45 \)
69. \( s = 9.87, r = 1.12 \)

In problems 70 – 71, evaluate the expression used to calculate the desired nozzle pressure of a fire hose: \( p + g - f \) for each given values \( p \), \( g \), and \( f \). Round each answer to the nearest hundredth.

70. \( p = 25.78, g = 200, f = 42.8 \)
71. \( p = 15.895, g = 174.32, f = 88.322 \)

In problems 72 – 73, evaluate the expression used to calculate the desired nozzle pressure of a fire hose: \( p - L - f \) for each given values \( p \), \( L \), and \( f \). Round each answer to the nearest hundredth.

72. \( p = 34.7, L = 150, f = 76.8 \)
73. \( p = 56.876, L = 99.876, f = 42.3 \)

In problems 74 – 75, evaluate the expression used to calculate the fuel moisture content percentage: \( 100 \left( \frac{w - d}{d} \right) \) for each given values \( w \) and \( d \). Round each answer to the nearest hundredth.

74. \( w = 345, d = 204 \)
75. \( w = 234, d = 132 \)

3. APPLICATIONS FOR ACCOUNTING

FUN FACT: In business, **Net** = **Revenue** – **Cost**. If the net is positive, then a profit is made, otherwise it is a loss. If net = 0, then the business **breaks even**.

In problems 76 – 78, use the expression for net (revenue – cost) to answer the questions.

76. A company produces two different baseball bats, a tapered handle bat and a non-tapered handle bat. If \( t \) represents the number of tapered handled bats and \( n \) represents the number of non-tapered handle bats, then \( 215t + 145n + 100 \) describes the revenue from sales of the two types of bats. The polynomial \( 140t + 110n + 345 \) describes the cost of producing the two types of bats.
   a) Write an expression for the net in simplest form.
b) If the company sells 120 non-tapered handled bats and 106 tapered bats in one month, what would be the net profit or loss?

77. Suppose the expression $9w + 45$ describes the revenue in dollars a tiling business receives for tiling a rectangular room, where $l$ represents the length of the room and $w$ represents the width. The expression $3lw + t$ describes the tiling business's cost in dollars for tiling a rectangular room, where $t$ represents the number of tiles required.

a) Write an expression for the net in simplest form.

b) If the tiling business gets a job to tile a 12-foot-wide by 14-foot-long room and 672 tiles are used. Calculate the profit.

78. Suppose you run a photography business selling three different photo packages in three sizes: small, medium, and large. You sell the small packages for $5, medium for $9, and large for $15. The small packages cost you $2 each to make, medium $4 each, and large $7 each.

a) Write an expression that describes the revenue that you will receive from the sale of all three packages.

Let $s =$ number of small packages, $m =$ number of medium packages, and $l =$ number of large packages.

b) Write an expression that describes the cost to produce all three packages.

Let $s =$ number of small packages, $m =$ number of medium packages, and $l =$ number of large packages.

c) Write an expression for your net in simplest form using your expressions in (a) and (b).

d) If in one day you sell 6 small, 9 medium, and 3 large packages, what would be your net? Is it a profit or loss?

4. APPLICATIONS FOR CULINARY ARTS

In problems 79 – 82, evaluate the expression used to calculate the edible portion cost: \( \frac{C}{r} \) for each given values $C$ and $r$. Round each answer to the nearest hundredth.

79. $C = 5.95$, $r = 0.234$
80. $C = 10.32$, $r = 0.33$
81. $r = 0.45$, $C = 9.87$
82. $r = 0.345$, $C = 12.92$

In problems 83 – 84, use the expression for net (revenue – cost) to answer the questions.

83. A restaurant has fixed costs of $200,000 and variable costs of 70% of the check average. The check average is $12.50.

a) Write an expression that describes the revenue the restaurant receives from each meal served. Let $x =$ # of meals served.

b) Write an expression that describes the restaurants total costs. Let $x =$ # of meals served.

c) Write an expression for the restaurants net. Let $x =$ # of meals served.

d) If this restaurant sold 54,348 meals, what would the net be? Is it a profit or loss?
84. Given a restaurant sales of $180,000, variable costs of $68,000 and fixed costs of $95,000. What is the profit?

85. As the owner of a restaurant in Vail, you are trying to determine if you should stay open next week. Given the following information,
   a) Write an expression for the total revenue. Let x = amount of sales (in dollars).
   b) Write an expression for the total costs. Let x = amount of sales (in dollars).
   c) Use a sales forecast of $3,500 and your expression for net to calculate the net (profit or loss) and determine if you should stay open.
   Salaries $1900
   Utilities $300
   Variable Costs 48% of sales

In problems 86 – 87, answer the questions.

86. Given fixed costs of $100,000, and a variable cost rate of 40% of food sales, write an expression for the total costs.

87. If a restaurant received $250,000 in sales from problem (86), what is the restaurants total costs?

5. APPLICATIONS FOR EARLY CHILDHOOD EDUCATION

In problems 88 – 91, evaluate the expression used to calculate the area of a triangle: \( \frac{bh}{2} \) for each given values b and h.

88. b = 12, h = 8
89. b = 13, h = 5
90. b = 87, h = 13
91. b = 64, h = 128

In problems 92 – 95, evaluate the expression used to calculate the area of a trapezoid:
\( \frac{1}{2}h(a + b) \) for each given values a, b and h. Round each answer to the nearest hundredth

92. a = 23.23, b = 54.5, h = 31
93. b = 95.654, h = 48, a = 24.73
94. h = 49 ½, a = 22 ¾, b = 58
95. h = 21 ¾, a = 95 ½, b = 73 ¾
In problems 96 – 99, evaluate the expression used to calculate the surface area of a cone: 
\[ \pi r^2 + \pi rL \] for each given values \( r \), \( L \), and \( \pi \approx 3.14 \). Round each answer to the nearest hundredth.

96. \( r = 3 \), \( L = 7.5 \)
97. \( r = 4.75 \), \( L = 6 \)
98. \( L = 23 \frac{1}{2} \), \( r = 15.7 \)
99. \( L = 24.2 \), \( r = 9 \frac{3}{4} \)

In problem 100, answer the questions.

100. For the rectangle to the right,
    a) Write an expression for the perimeter.
    b) Write an expression for the area.
    c) Evaluate your answer in b) when \( x = 3 \).

\[ \begin{array}{c}
4x + 8 \\
2x - 3
\end{array} \]

6. APPLICATIONS FOR GRAPHIC DESIGN/PROFESSIONAL PHOTOGRAPHY

In problems 101 – 104, evaluate the expression used to calculate the area of a circle: \( \pi r^2 \) for each given value \( r \) and \( \pi \approx 3.14 \). Round each answer to the nearest hundredth.

101. \( r = 30 \)
102. \( r = 24 \)
103. \( r = 19.5 \)
104. \( r = 37 \frac{3}{4} \)

In problems 105 – 108, evaluate the expression used for finding the volume of a cylinder: 
\[ 2\pi r^2 + 2\pi rh \] for each given values \( r \), \( h \) and \( \pi \approx 3.14 \). Round each answer to the nearest hundredth.

105. \( r = 47.98 \), \( h = 12 \)
106. \( r = 31 \), \( h = 98.43 \)
107. \( h = 39 \frac{1}{2} \), \( r = 26 \frac{3}{4} \)
108. \( h = 21 \frac{1}{4} \), \( r = 65 \frac{1}{2} \)

In problems 109 – 112, evaluate the expression used to calculate the volume of a sphere:
\[ \frac{4\pi r^3}{3} \] for each given value \( r \) and \( \pi \approx 3.14 \). Round each answer to the nearest hundredth.

109. \( r = 6 \)
110. \( r = 8.432 \)
111. \( r = 34.5 \)
112. \( r = 15 \frac{3}{4} \)

In problem 113, answer the questions.

113. For the square to the right,
   a) Write an expression for the perimeter.
   b) Write an expression for the area.
   c) Evaluate your answer in b) when \( x = 4.7 \).

In problems 114 – 115, evaluate the expression used to calculate the cost of ink: \( w \cdot p + c \) for each given values \( w, p \) and \( c \). Round each answer to the nearest hundredth.

114. \( w = 1.85, p = 15.35, c = 5.5 \)
115. \( w = 2.34, p = 20.45, c = 8.25 \)

In problems 116 – 119, evaluate the expression used to calculate the bellows factor: \( \frac{E^2}{L^2} \) for each given values \( E \) and \( L \). Round each answer to the nearest thousandth.

116. \( E = 12.5, L = 8.268 \)
117. \( E = 15.25, L = 9.543 \)
118. \( L = 11.811, E = 16.25 \)
119. \( L = 4.32, E = 10.78 \)

In problems 120 – 123, use the “log” function on your calculator to evaluate the expression used to calculate the f-stop adjustment: \( \log \frac{B}{0.3} \) for each given value \( B \). Round each answer to the nearest thousandth.

120. \( B = 1.547 \)
121. \( B = 4.678 \)
122. \( B = 3.899 \)
123. \( B = 5.987 \)

In problems 124 – 127, evaluate the expression used to calculate the normal focal length: \( \sqrt{b^2 + c^2} \) for each given values \( b \) and \( c \). Round each answer to the nearest tenth.

124. \( b = 4.5, c = 3.4 \)
125. \( b = 5.3, c = 4 \)
126. \( b = 6.4, c = 4.8 \)
127. \( b = 22.7, c = 15.1 \)
In problem 128, use the expression for net (revenue – cost) to answer the questions.

128. Alysha photographs portraits and sells them in three sizes, small (3 by 5), medium (5 by 7), and large (8 by 10). She receives $5 for a small, $8 for a medium, and $12 for a large photo. It costs her $2 to print a small photo, $3 for a medium, and $5 for each large photo.
   a) Write an expression that describes the revenue she receives from the sale of all three photo sizes.
      Let \( s \) = number of small photos, \( m \) = number of medium photos, and \( l \) = number of large photos.
   b) Write an expression that describes her cost to produce all three photo sizes.
      Let \( s \) = number of small photos, \( m \) = number of medium photos, and \( l \) = number of large photos.
   c) Write an expression for her net in simplest form.
   d) If in one day she sells 4 small, 6 medium, and 2 large photos, what would be her net profit or loss?

7. APPLICATIONS FOR INTEGRATED ENERGY TECHNOLOGY

In problems 129 – 132, evaluate the expression used to calculate voltage: \( i \cdot r \) for each given values \( i \) and \( r \).

129. \( i = -9, r = 7 \)
130. \( i = -12, r = 100 \)
131. \( i = -44, r = 5 \)
132. \( i = -32, r = 8 \)

In problems 133 – 136, evaluate the expression used to calculate heat transferred: \( mC(T - t) \) for each given values \( m \), \( C \), \( T \), and \( t \). Round each answer to the nearest tenth.

133. \( m = 54, C = 1, T = 100, t = 72.3 \)
134. \( m = 60, C = 1, T = 98.7, t = 65.4 \)
135. \( m = 72.3, C = 0.86, T = 90, t = 68 \)
136. \( m = 88.5, C = 0.86, T = 95, t = 60 \)

In problems 137 – 140, evaluate the expression used to calculate electric energy: \( \frac{V^2t}{r} \) for each given values \( V \), \( t \), and \( r \).

137. \( V = -63, r = 7, t = 24 \)
138. \( V = 1200, r = 100, t = 8 \)
139. \( V = -220, r = 5, t = 1.5 \)
140. \( V = 256, r = 8, t = 7/24 \)
In problems 141 – 144, evaluate the expression used to calculate the array size of a solar PV system: \[ \frac{4kr}{1095s} \] for each given values k, r, and s.

141. \( k = 9000, \ r = 0.5, \ s = 4.5 \)
142. \( k = 4570, \ r = 0.8, \ s = 3.6 \)
143. \( k = 12342, \ r = 0.95, \ s = 8.2 \)
144. \( k = 7822, \ r = 0.78, \ s = 6.6 \)

8. APPLICATIONS FOR PROCESS TECHNOLOGY

In problems 145 – 148, evaluate the expression used to calculate the initial height of a falling object: \( \frac{1}{2}gt^2 \) for each given values g and t.

145. \( g = 32, \ t = 5.4 \)
146. \( g = 32.174, \ t = 8.32 \)
147. \( g = 10, \ t = 4.7 \)
148. \( g = 9.80665, \ t = 9.7 \)

In problems 149 – 152, evaluate the expression used to calculate the time to impact of a falling object: \( \sqrt{\frac{2h}{g}} \) for each given values g and h.

149. \( g = 32, \ h = 235.7 \)
150. \( g = 32.174, \ h = 26.2 \)
151. \( g = 10, \ h = 42.3 \)
152. \( g = 9.80665, \ h = 152.8 \)

In problems 153 – 156, evaluate the expression used to calculate the velocity at impact of a falling object: \( 8\sqrt{h} \) for each given value h.

153. \( h = 235.7 \)
154. \( h = 26.2 \)
155. \( h = 183.3 \)
156. \( h = 10.7 \)
In problems 157 – 160, evaluate the expression used to calculate the hydraulic gradient between two wells: \( \frac{h_2 - h_1}{r} \) for each given values \( h_2, h_1, \) and \( r \).

157. \( h_2 = 13.2, h_1 = 12.7, r = 350 \)
158. \( h_2 = 10.8, h_1 = 9.9, r = 440 \)
159. \( h_2 = 8.7, h_1 = 8.5, r = 560 \)
160. \( h_2 = 11.7, h_1 = 9.8, r = 1250 \)

In problems 161 – 164, evaluate the expression used to calculate the arc length of a circle: \( \frac{\pi r \theta}{180} \) for each given values \( \pi \approx 3.14, r, \) and \( \theta \).

161. \( r = 16, \theta = 72 \)
162. \( r = 7, \theta = 156 \)
163. \( r = 5.8, \theta = 6.7 \)
164. \( r = 9.7, \theta = 345 \)

In problems 165 – 168, evaluate the expression used to calculate the area of a sector of a circle: \( \frac{\pi r^2 \theta}{360} \) for each given values \( \pi \approx 3.14, r, \) and \( \theta \).

165. \( r = 16, \theta = 72 \)
166. \( r = 7, \theta = 156 \)
167. \( r = 5.8, \theta = 6.7 \)
168. \( r = 9.7, \theta = 345 \)

In problems 169 – 172, evaluate the expression used to calculate the volume of an ellipse: \( \pi abh \) for each given values \( \pi \approx 3.14, a, b, \) and \( h \).

169. \( a = 12, b = 16, h = 7 \)
170. \( a = 9, b = 144, h = 8 \)
171. \( a = 14.8, b = 9.7, h = 12.7 \)
172. \( a = 18.9, b = 10.2, h = 17.6 \)

In problems 173 – 176, evaluate the expression used to calculate the surface area of an elliptical tank: \( 2\pi \left(ab + h\sqrt{\frac{a^2 + b^2}{2}}\right) \) for each given values \( \pi \approx 3.14, a, b, \) and \( h \).

173. \( a = 12, b = 16, h = 7 \)
174. \( a = 9, b = 144, h = 8 \)
175. \( a = 14.8, b = 9.7, h = 12.7 \)
176. \( a = 18.9, b = 10.2, h = 17.6 \)
9. APPLICATIONS FOR SKI AREA OPERATIONS

In problems 177 – 180, evaluate the expression used to calculate the blade capacity of a standard front-loader: \( \frac{lwh}{2} \) for each given values \( l \), \( w \), and \( h \).

177. \( l = 12, w = 16, h = 7 \)
178. \( l = 9, w = 144, h = 8 \)
179. \( l = 14.8, w = 9.7, h = 12.7 \)
180. \( l = 18.9, w = 10.2, h = 17.6 \)

In problems 181 – 184, evaluate the expression used to calculate the passengers per hour on a ski lift: \( \frac{3600p}{i} \) for each given values \( p \), and \( i \).

181. \( p = 2, i = 7 \)
182. \( p = 1, i = 8 \)
183. \( p = 3, i = 5 \)
184. \( p = 4, i = 6 \)

In problems 185 – 188, evaluate the expression used to calculate the vertical transport feet per hour: \( \frac{3600p(h_2 - h_1)}{i} \) for each given values \( p \), \( h_2 \), \( h_1 \), and \( i \).

185. \( p = 2, h_2 = 10,524, h_1 = 7450, i = 7 \)
186. \( p = 1, h_2 = 9876, h_1 = 6850, i = 8 \)
187. \( p = 3, h_2 = 11,654, h_1 = 8892, i = 5 \)
188. \( p = 4, h_2 = 10,320, h_1 = 7005, i = 6 \)

In problems 189 – 192, evaluate the expression used to calculate the skiers at one time: \( \frac{3600pt(h_2 - h_1)}{id} \) for each given values \( p \), \( t \), \( h_2 \), \( h_1 \), \( i \), and \( d \).

189. \( p = 2, t = 8, h_2 = 10,524, h_1 = 7450, i = 7, d = 2000 \)
190. \( p = 1, t = 10, h_2 = 9876, h_1 = 6850, i = 8, d = 8000 \)
191. \( p = 3, t = 12, h_2 = 11,654, h_1 = 8892, i = 5, d = 20,000 \)
192. \( p = 4, t = 7, h_2 = 10,320, h_1 = 7005, i = 6, d = 10,500 \)
Solutions to Module IV:
1. -17  2. -18  3. 6  4. 0  5. 274  6. $35b^3 - 13b^2 + 5b$  7. $-2x^2 - 3x + 2$  8. -3a + 10b
9. $3x^2 - 6xy + 9y^2$  10. $-5.1a^2bc + 3.9ab^2c$  11. $\frac{-23}{30}a + \frac{43}{12}b - \frac{9}{4}c$  12. -4a + 8b
13. $14x - 4xy$  14. $7x^2 - 3xy + 3y^2$  15. $12a + 10b - 3c$  16. -13mn
17. $-x^5 - 9x^4 - 6x^3 - 9x^2 + 16$  18. $2t^3 - 13t^2 - 6t^2 + t - 5$  19. $-x^2 + 14xy + 15y^2$
20. -17a + 10b  21. $46a + 180b - 120$  22. $-36a^2 - 20ab + 18a + 30b$
23. $-185a^2 + 55ab - 6$  24. -8a + 9  25. a) $20x - 2$  b) $21x^2 + x - 2$  c) 528 square units
26. False  27. True  28. False  29. $4.5 \times 10^{-7}$  30. $1.504 \times 10^{-5}$  31. Approximately
1.15385 $\times 10^{12}$ times as long  32. $x^9$  33. $(x + 4)^8$  34. $3^6$  35. $125x^3$  36. $-27y^6$  37. $200x^{19}$
38. $-192x^{39}$  39. $\frac{1}{t^4}$  40. 1  41. $\frac{a^2}{9b^6}$  42. $\frac{c^3}{a^6b^3}$  43. $y^{7x}$  44. $625x^{12}$  45. $-\frac{243x^{10}}{32y^{15}}$
46. $81x^4(x + 1)^4$  47. $x^{3k + 4}$  48. $36x^{14}$  49. $\frac{d^9b^{16}c^8}{a^3}$  50. $\frac{27}{8a^1b^6}$  51. $\frac{9y^7}{x^8}$  52. $x^2 + 6x + 9$
53. $25x^2 - 20xy + 4y^2$  54. $a^2 - b^2$  55. $4y^2 - 1 - 56$  54. $x^2 + 9y^2$
57. $10x^6y - 13x^4y^3 - 3x^2y^5$  58. $27x^3 - 125$  59. $45a^2 - 162a - 143$  60. $9xy^2z^3$
61. $-3xy + x + 2y$  62. 1.60  63. 1.87  64. 2.15  65. 2.25  66. -170.88  67. -28.75  68. 5.44
69. 8.81  70. 182.98  71. 101.89  72. -192.10  73. -85.3  74. 69.12  75. 77.27
76. a) $75t + 35n - 245$  b) $\$11,905 profit  77. a) $6lw + 45 - t$  b) $\$381  78. a) $R = 5s + 9m + 15l$
b) $C = 2s + 4m + 7l$  c) $N = 3s + 5m + 8l$  d) $\$87 profit  79. 25.43  80. 31.27  81. 21.93
82. 37.45  83. a) $12.50x$  b) $8.75x + 200,000$  c) $3.75x - 200,000$  d) $\$3805; profit 84. $\$17,000$
a) $-0.4x + 2200$  c) $\$380 loss. You should close. 86. 0.4x + 100,000  87. $\$200,000$
88. 48  89. 32.5  90. 565.5  91. 4096  92. 1204.82  93. 2889.22  94. 1986.19  95. 1835.16
96. 98.91  97. 160.34  98. 1932.38  99. 1039.38  100. a) $12x + 10$  b) $8x^2 + 4x - 24$  c) 60
square units  101. 2826  102. 1808.64  103. 1193.99  104. 4474.70  105. 18.072.84
106. 25.197.43  107. 10.838.89  108. 35.889.42  109. 904.32  110. 2509.92  111. 171,919.71
112. 16,357.24  113. a) $16x - 36$  b) $16x^2 - 72x + 81$  c) 96.04 square units  114. 33.90
115. 56.10  116. 2.286  117. 2.554  118. 1.893  119. 6.227  120. 0.632  121. 2.234
122. 1.970  123. 2.591  124. 5.6  125. 6.6  126. 8  127. 27.3  128. a) $5s + 8m + 12l$
b) $2s + 3m + 5l$  c) $3s + 5m + 7l$  d) $\$56 profit  129. -63  130. -120  131. -220  132. -256
133. 1495.8  134. 1998  135. 1367.9  136. 2663.9  137. 13,608  138. 115,200  139. 14,520
140. 2389.3  141. 3.65  142. 3.71  143. 5.22  144. 3.38  145. 466.56  146. 1113.58
147. 110.45  148. 461.35  149. 3.84  150. 1.28  151. 2.90  152. 5.58  153. 122.82  154. 40.95
155. 108.31  156. 26.17  157. 0.00143  158. 0.00205  159. 0.000357  160. 0.00152
161. 20.096  162. 19.049  163. 0.678  164. 58.378  165. 160.768  166. 66.673  167. 1.966
168. 283.132  169. 4220.16  170. 32555.52  171. 5724.886  172. 10653.794  173. 1827.448
174. 13264.468  175. 1899.511  176. 2889.174  177. 672  178. 5184  179. 911.606
180. 1696.464  181. 1028.57  182. 450  183. 2160  184. 2400  185. 3,161,828.57
186. 1,361,700  187. 5,965,920  188. 7,956,000  189. 12647.31  190. 1702.125
191. 3579.552  192. 5304
Module V
Factoring Polynomials: An Introduction

In Module IV we learned how to evaluate, simplify, and perform operations on polynomial expressions. In this module we will begin our journey in factoring very basic types of polynomials. Specifically, we will learn how to factor out common factors; factor by grouping; factor binomials that fall into the category of difference of squares; and factor trinomials with a leading coefficient of 1. Since this module is designed to be an introduction to factoring, we will be leaving out some factoring strategies, such as factoring binomials that fall into the category of difference of cubes or sum of cubes and factoring trinomials with a leading coefficient that is not equal to 1.

The ability to factor is, in my opinion, the single most important tool that a mathematics student can possess. The need to factor appears over and over throughout many different mathematics courses and in many different math course competencies. If unable to perform correctly, a student may see increasing difficulty in the math course they are attempting to pass. Please, please, please, take this module seriously!

We will begin our journey into factoring with its definition.

I. Definition of Factoring

In mathematics, many exercises look very much the same. It’s the directions that help us determine what it is we need to do with the problem. It is easy to mistake “evaluate”, with “simplify”, or “solve”, or “factor”, since the expressions in the problems themselves all look identical. For example, consider the following exercises, all of which appear in the same algebra class:

1. Evaluate $3x^2 + 6x + x + 2$ for $x = -2$
2. Simplify: $3x^2 + 6x + x + 2$
3. Solve for $x$: $3x^2 + 6x + x + 2 = 0$
4. Factor: $3x^2 + 6x + x + 2$

In completing math exercises, often times we look for visual cues that help us determine what it is we need to do with the problem. As you can see in the examples above, it would be easy for us to mistake one direction with the next. Just so we are clear, evaluate in problem (1) means to find the value for the expression when $x = -2$ is substituted. Simplify in problem (2) means to write the expression in its simplest form, that is, combine like terms. Solve for $x$ in problem (3) means to find values for $x$ that makes the equation true. **Factor** in problem (4) means to **write the polynomial as a product** (multiplication).
Let’s look at ways to write an expression as a product. Consider the number “12”. If we were asked to factor 12, there would be many different ways to write 12 as a product:

\[ 12 = 1 \cdot 12 \text{ or } 12 = 2 \cdot 6 \text{ or } 12 = 3 \cdot 4 \text{ or } 12 = 2 \cdot 2 \cdot 3 \]

Only the factorization \( 12 = 2 \cdot 2 \cdot 3 \) would be considered to be in prime factored form. That is written as a product of prime numbers (a number that is divisible by only 1 and itself). This factorization is unique, that is, there is only one way for us to prime factor 12. The Fundamental Theorem of Arithmetic gives us the uniqueness of prime factorization of integers greater than 1. It is our goal when factoring polynomials “completely”, to factor the polynomial until each factor is “prime”. That is, cannot be factored further.

Now that we have a definition, consider some of the problems that we were asked to multiply in module IV:

1) \( xy^2 \left( x^2 + 4y \right) = x^3y^2 + 4xy^3 \)
2) \( (3x - 4)(x - 5) = 3x^2 - 19x + 20 \)
3) \( (2x - 1)(2x - 1) = 4x^2 - 4x + 1 \)
4) \( (4x + 9)(4x - 9) = 16x^2 - 81 \)

Notice how the left side of the equality is in factored form, and the right side is in expanded form. When factoring, we want to take the expanded form on the right and reverse the process to write the factored form on the left.

We will begin by looking at factoring out “what’s in common”, better known as the Greatest Common Factor (GCF).

**II. Factoring the Greatest Common Factor**


The Greatest Common Factor of two or more terms is the “largest” factor that divides evenly into each of the terms. For example if the terms were numerical, like 16 and 24, then the greatest common factor of 16 and 24 is 8, since 8 is the largest number that divides evenly into 16 and 24. If the terms involved variables raised to exponents, for example \( x^2 \) and \( x^3 \), then \( x^2 \) is the largest variable expression that divides evenly into \( x^2 \) and \( x^3 \). Putting this together, suppose we wanted to find the Greatest Common Factor of \( 16x^2 \) and \( 24x^3 \), then we would find that it is \( 8x^2 \) since \( 8x^2 \) is the “largest” factor that divides evenly into \( 16x^2 \) and \( 24x^3 \).
Factoring out the Greatest Common Factor (GCF):
1) Consider each term that makes up the polynomial and factor each into a product of prime numbers and variables.
2) Find the greatest common factor of those terms.
3) Write each term as a product of the greatest common factor and the remaining factors.
4) Write the expression in factored form.

Example 1: Factor completely: $6x^2y^2 - 9xy^3$

Solution to Example 1:
If we wanted to factor out the GCF, we would first consider the two terms that make up the polynomial, $6x^2y^2$ and $-9xy^3$ and factor each into prime numbers and variables:

1) $6x^2y^2 = 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y$

2) $-9xy^3 = -1 \cdot 3 \cdot 3 \cdot x \cdot y \cdot y \cdot y$

Then we would find the greatest common factor of those terms:

3) $3x^2y^2 = \frac{6x^2y^2}{2} = \frac{-9xy^3}{3}$

Next, we would write each term as a product of the greatest common factor and the remaining factors:

4) $3x^2y^2 \cdot 2x - 3x^2y^2 \cdot 3y$

Finally, we would factor out the greatest common factor and write in factored form:

5) $3xy^2(2x - 3y)$

Answer: $3xy^2(2x - 3y)$

To check our answer we simply multiply the polynomials using the distributive property to determine if we get back to the original polynomial:

CHECK:

$3xy^2(2x - 3y) = 6x^2y^2 - 9xy^3$
III. Factoring Polynomial Common Factors


Suppose we have a polynomial of the following form: \(2xy(3y + 1) - 3xz(3y + 1)\). Notice that we could split this into two expressions: \(2xy(3y + 1)\) and \(-3xz(3y + 1)\). Also notice that each of these two expressions have \((3y + 1)\) in common. If we wanted to factor \(2xy(3y + 1) - 3xz(3y + 1)\) completely, we could treat each “common expression” as a common factor. The greatest common factor of \(2xy(3y + 1) - 3xz(3y + 1)\) then would be \((3y + 1)\). After factoring \((3y + 1)\) out we get: \(x(3y + 1)(2y - 3z)\).

Example 2: Factor completely: \(x^2(2x + 3) + 4(2x + 3)\)

Solution to Example 2:
The two expressions to consider for the common factor are \(x^2(2x + 3)\) and \(4(2x + 3)\). Notice that \((2x + 3)\) is common to both, so we will treat this as a common factor, and there is no other common factor. Factoring this out we get: \((2x + 3)(x^2 + 4)\)

Answer: \((2x + 3)(x^2 + 4)\)

IV. Factoring by Grouping


Consider our previous example 2: \(x^2(2x + 3) + 4(2x + 3)\). Another way to write this, using the distributive property, would be \(2x^3 + 3x^2 + 8x + 12\). Suppose now we were asked to factor \(2x^3 + 3x^2 + 8x + 12\) completely. Because this polynomial has 4 terms we would need to use a method called Factoring by Grouping. That is, we would group the first two terms together and factor out the greatest common factor, and the last two terms together and factor out the greatest common factor. It would resemble something like this:

\[
\underbrace{2x^3 + 3x^2}_{x^2(2x + 3)} + \underbrace{8x + 12}_{4(2x + 3)}
\]
As seen in example 2, we can continue to factor the expression \( x^2(2x + 3) + 4(2x + 3) \) to \( (2x + 3)(x^2 + 4) \).

**Example 3:** Factor completely: \( x^3 + 3x^2 + 2x + 6 \)

Solution to Example 3:
Noticing 4 terms in the polynomial, we will use factoring by grouping. The first two terms factor to \( x^2(x + 3) \), and the last two terms factor to \( 2(x + 3) \). Notice that \( x + 3 \) is common to both, so we will treat this as a common factor, and there is no other common factor. Factoring this out we get:

\[
(x + 3)(x^2 + 2)
\]

**Answer:** \( (x + 3)(x^2 + 2) \)

We would use factoring by grouping on polynomials that have 4 or more terms. Also, it is important to note that if we don’t have a common polynomial factor, then factoring by grouping will not work.

The following example is a case where factoring by grouping doesn’t work.

**Example 4:** Factor completely: \( x^3 + 5x^2 + 7x + 49 \)

Solution to Example 4:
\( x^3 + 5x^2 + 7x + 49 = x^2(x + 5) + 7(x + 7) \). Since the factors in the parentheses are different, we cannot continue factoring them out. Notice also that if we were to move terms around we still will not get common factors in the parentheses: \( x^3 + 7x + 5x^2 + 49 = x(x^2 + 7) + 1(5x^2 + 49) \). When this happens we say that the polynomial is **not factorable** or **prime**.

**Answer:** not factorable or prime.

**V. Factoring Binomials by Difference of Squares**


There are only 3 ways that we can factor a binomial (if factorable). The binomial is factorable when:

1) The binomial is a **Difference of Squares**;
2) The binomial is a **Difference of Cubes**; or
3) The binomial is a **Sum of Cubes**
We will introduce only the **Difference of Squares** method for factoring binomials in this module.

Difference of squares literally means to subtract two perfect squares. For instance, 25 and 9 are perfect squares since $25 = 5^2$ and $9 = 3^2$. If we were to subtract $25 - 9$, we have a difference of squares. To write $25 - 9$ as a product we could write $25 - 9 = 5^2 - 3^2 = (5 + 3)(5 - 3)$. This makes sense, since $25 - 9 = 16 = (8)(2) = (5 + 3)(5 - 3)$. As it turns out, anytime we have any expression of the form $A^2 - B^2$, we may factor it into $(A + B)(A - B)$.

**Difference of Squares:**

$$A^2 - B^2 = (A + B)(A - B)$$

Note: There is no “sum of squares” factorization. A common mistake is to think that $A^2 + B^2$ factors to $(A + B)(A + B)$ or $(A + B)^2$. We’ve seen in Module IV that $(A + B)^2 = A^2 + 2AB + B^2 \neq A^2 + B^2$

The key to factoring binomials that are difference of squares is to determine the “A” and “B” value, then substitute those values into the formula.

**Example 5:** Factor the following completely:

a) $x^2 - 4$

b) $16x^2 - 1$

c) $-64 + x^2$

d) $25x^2 - 49y^2$

e) $4x^2 + 9$

f) $x^4 - 81$

Solutions to Example 5:

a) $x^2 - 4 = (x)^2 - (2)^2$. $A = x$, and $B = 2$. So the factorization is $(x + 2)(x - 2)$.

**Answer:** $(x + 2)(x - 2)$

b) $16x^2 - 1 = (4x)^2 - (1)^2$. $A = 4x$, and $B = 1$. So the factorization is $(4x + 1)(4x - 1)$.

**Answer:** $(4x + 1)(4x - 1)$

c) $-64 + x^2$ should be put in order: $x^2 - 64$. $x^2 - 64 = (x)^2 - (8)^2$. $A = x$, and $B = 8$. So the factorization is $(x + 8)(x - 8)$.

**Answer:** $(x + 8)(x - 8)$

d) $25x^2 - 49y^2 = (5x)^2 - (7y)^2$. $A = 5x$, and $B = 7y$. So the factorization is $(5x + 7y)(5x - 7y)$.

**Answer:** $(5x + 7y)(5x - 7y)$

e) $4x^2 + 9$. Although we two perfect squares, this is a sum of squares, which is not factorable.

**Answer:** Not factorable or prime
f) \( x^4 - 81 = (x^2)^2 - (9)^2 \). A = \( x^2 \), and B = 9. So the factorization is \( (x^2 + 9)(x^2 - 9) \). However, this is not our final answer since \( x^2 - 9 \) can be factored further by difference of squares, i.e. \( (x + 3)(x - 3) \). So our final factorization is \( (x^2 + 9)(x + 3)(x - 3) \), where \( x^2 + 9 \) is not factorable any further since it is a sum of squares.

Answer: \( (x^2 + 9)(x + 3)(x - 3) \)

VI. Factoring Trinomials with Leading Coefficient = 1


A **trinomial with leading coefficient = 1** is any trinomial of the form \( x^2 + bx + c \). There are a few different approaches to factoring these types of trinomials. The approach that we will use is called the “**Trial and Error**” approach. This approach utilizes a process of elimination combined with a guess and check strategy.

**Factoring Trinomials of the Form \( x^2 + bx + c \):**

1. First, make sure the polynomial is in the proper order: \( x^2 + bx + c \). ORDER MATTERS!
2. Essentially, we know that a trinomial, if factorable, will factor into two binomials, so we start with writing the parentheses: \( ( ) ( ) \).
3. Next, we know that the only way that we can multiply the first two terms together and get the middle term “\( bx \)” is to have \( (x \ ) (x \ ) \).
4. Here’s the hard part. We need to find two numbers that multiply to give us “\( c \)” and add or subtract to give us “\( b \)”. To begin to locate these numbers, we list all of the factor pairs of “\( c \)”.
5. From the factor pairs of “\( c \)” that we listed, locate the pair (we’ll call them \#1 and \#2) that adds or subtracts to get “\( b \)” and put those numbers in the parentheses: \( (x \ #1)(x \ #2) \).
6. Finally, we determine the signs (+ or −) that we will put in the parentheses between our numbers. Here’s where the trial and error comes in to play. Try the signs, and if they don’t work, try a different combination. If you’ve exhausted all your combinations and factor pairs, and you still don’t get a correct factorization, then the trinomial is prime.

**Determining the signs (+ or −):**

<table>
<thead>
<tr>
<th>Signs in Trinomial (assume ( b, c &gt; 0 ))</th>
<th>Factorization</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 + bx + c )</td>
<td>( (+)(+)(+)(+) )</td>
</tr>
<tr>
<td>( x^2 - bx + c )</td>
<td>( (-)(-)(-)(-) )</td>
</tr>
<tr>
<td>( x^2 + bx - c )</td>
<td>( (+)(-)(+)(-) )</td>
</tr>
<tr>
<td>( x^2 - bx - c )</td>
<td>( (+)(-)(-)(+) )</td>
</tr>
</tbody>
</table>
**Example 6:** Factor the following completely:

a) \( x^2 + 7x + 6 \)

b) \( x^2 - 12x + 35 \)

c) \( 4x + x^2 + 4 \)

d) \( x^2 - 4x - 21 \)

e) \( x^2 + 7x - 30 \)

f) \( x^2 + 5x + 3 \)

**Solutions to Example 6:**

a) \( x^2 + 7x + 6 \). Start with parentheses: \((x \quad)(x \quad)\). Determine factor pairs of 6 that add or subtract to get 7: 1 ∙ 6 or 2 ∙ 3. 1 and 6 add to get 7, so we’ll use those: \((x + 1)(x + 6)\). Determine the signs: Since the trinomial is + and +, then the factorization should be + and +: \((x + 1)(x + 6)\). Check this answer by multiplying: \(x^2 + 6x + x + 6 = x^2 + 7x + 6\). This works!  
**Answer:** \((x + 1)(x + 6)\)

b) \( x^2 - 12x + 35 \). Start with parentheses: \((x \quad)(x \quad)\). Determine factor pairs of 35 that add or subtract to get 12: 1 ∙ 35 or 5 ∙ 7. 5 and 7 add to get 12, so we’ll use those: \((x - 5)(x - 7)\). Determine the signs: Since the trinomial is – and + (in that order), then the factorization should be – and – : \((x - 5)(x - 7)\). Check this answer by multiplying: \(x^2 - 7x - 5x + 35 = x^2 - 12x + 35\). This works!  
**Answer:** \((x - 5)(x - 7)\)

c) \( 4x + x^2 + 4 \). This is not in order, so we will write in descending order: \(x^2 + 4x + 4\). Start with parentheses: \((x \quad)(x \quad)\). Determine factor pairs of 4 that add or subtract to get 4: 1 ∙ 4 or 2 ∙ 2. 2 and 2 add to get 4, so we’ll use those: \((x + 2)(x + 2)\). Determine the signs: Since the trinomial is + and +, then the factorization should be + and +: \((x + 2)(x + 2) = (x + 2)^2\). Check this answer by multiplying: \(x^2 + 2x + 2x + 4 = x^2 + 4x + 4\). This works!  
**Answer:** \((x + 2)(x + 2) = (x + 2)^2\)

d) \( x^2 - 4x - 21 \). Start with parentheses: \((x \quad)(x \quad)\). Determine factor pairs of 21 that add or subtract to get -4: 1 ∙ 21 or 3 ∙ 7. 3 and 7 subtract to get -4, so we’ll use those: \((x + 3)(x - 7)\). Determine the signs: Since the trinomial is – and –, then the factorization should be + and –, but the order is important since they are different signs: \((x + 3)(x - 7)\). Check this answer by multiplying: \(x^2 - 7x + 3x - 21 = x^2 - 4x - 21\). This works! Notice that if we chose the signs in a different order we would’ve gotten \(x^2 + 4x - 21\).  
**Answer:** \((x + 3)(x - 7)\)
e) \(x^2 + 7x - 30\). Start with parentheses: \((x \quad)(x \quad)\). Determine factor pairs of 30 that add or subtract to get 7: 1 \cdot 30 or 2 \cdot 15 or 3 \cdot 10 or 5 \cdot 6. 3 and 10 subtract to get 7, so we’ll use those: \((x - 3)(x + 10)\). Determine the signs: Since the trinomial is + and – (in that order), then the factorization should be + and –: \((x - 3)(x + 10)\). Check this answer by multiplying: 
\[x^2 + 10x - 3x - 30 = x^2 + 7x - 30.\] This works!
Answer: \((x - 3)(x + 10)\)

f) \(x^2 + 5x + 3\). Start with parentheses: \((x \quad)(x \quad)\). Determine factor pairs of 3 that add or subtract to get 5: The only option is 1 \cdot 3, but this does not add or subtract to get 5. Because there are no other options, this trinomial is not factorable.
Answer: Not Factorable or Prime

Trinomials of the form \(x^{2n} + bx^n + c\) can be factored in a similar manner as \(x^2 + bx + c\), the only difference is \((x \quad)(x \quad)\) turns into \((x^n \quad)(x^n \quad)\). For example, to factor the trinomial \(x^4 + 5x^2 - 24\), we would first start with \((x^2 \quad)(x^2 \quad)\), then we look at factor pairs of 24 that add or subtract to be 5. The factor pairs we would use is 8 and 3 and the factorization would be \((x^2 + 8)(x^2 - 3)\).

VII. Factoring: A General Strategy

Here’s where we will put it all together. Some polynomial factorizations may require factoring a greatest common factor, some may require factoring by grouping, some may require factoring a difference of squares, some may require factoring trinomials, and some may require a combination of all these concepts. Should this happen, here’s how you organize your thoughts:

**Factoring Polynomials: A General Strategy:**

Goal for Factoring: Write a polynomial as a product in which each factor cannot be factored any further. If not factorable, then state so.

1. Factor out “what’s in common”, that is, the GCF of all terms.
2. Determine the number of terms in the polynomial.
   a. If there are four or more terms in the polynomial, then try factoring by grouping. For example, group the first two terms and factor the GCF, the group the remaining terms and factor out the GCF. If the polynomial is factorable then what remains in the parentheses should be IDENTICAL.
   b. If there are two terms (binomial), then try to factor a difference of squares:
      Note: There is no “Sum of Squares”, that is \(A^2 + B^2\) is NOT factorable.
   c. If there are three terms (trinomial with leading coefficient 1) then try factoring by “trial and error”
      i. Write the polynomial in descending order since ORDER MATTERS!
ii. We know that if a trinomial is factorable, then it must factor into two binomials so start with parentheses: ( )( )

iii. List the factor pairs of the last term (constant term).

iv. Locate the factor pairs of the last term that add (or subtract) to give you the coefficient of the middle term.

v. Determine the signs.

vi. Check your factorization. If it doesn’t work out, try a different combination and check again. Note: Not all trinomials are factorable!

3. Check each factor to determine if it can be factored further. If a factor can be factored further, repeat steps 1) and 2).

Example 6: Factor the following completely: \( x^2 y - 4x^4 y - 9x^3 y + 36x^2 y \)

Solution to Example 6:

1) Factor out the GCF of \( x^2 y \) to get \( x^2 y(x^3 - 4x^2 - 9x + 36) \)

2) Factor the polynomial with 4 terms by grouping:
\( x^2 y(x - 4)(x - 9) \)

3) Factor the difference of squares: \( x^2 y(x - 4)(x + 3)(x - 3) \)

4) Check to see if anything else is factorable: \( x^2 y(x - 4)(x + 3)(x - 3) \). Nothing can be factored further.

Answer: \( x^2 y(x - 4)(x + 3)(x - 3) \)

Homework Set:

In problems 1 – 6, factor the following completely. (Objective: Factor the GCF)

1. \( 2x^2 + 6x \)
2. \( 5x^3 + 10x^3 \)
3. \( 8x^2 - 4x - 20 \)
4. \( 16x^6 y^4 + 32x^4 y^3 - 48xy^2 \)
5. \( 5x^5 + 10x^2 - 8x \)
6. \( 27x^2 y^7 + 15x^2 y^3 - 3xy^2 \)
7. \( x^2(x + 3) + 2(x + 3) \)
8. \( 2x^2(4x - 3) - 5(4x - 3) \)
In problems 9 – 13, factor the following completely. (Objective: Factor by Grouping)

9. \( x^3 + 7x^2 - 2x - 14 \)
10. \( 18x^3 - 21x^2 + 30x - 35 \)
11. \( 7x^3 - 14x^2 - x + 2 \)
12. \( 2x^3 + 12x^2 - 5x - 30 \)
13. \( 20x^3 - 4x^2 - 25x + 5 \)

In problems 14 – 19, factor the following completely. (Objective: Factor Difference of Squares)

14. \( x^2 - 121 \)
15. \( x^2 - 81y^2 \)
16. \( -49 + x^2 \)
17. \( 25x^2 - y^6 \)
18. \( 9x^4 - 16y^2 \)
19. \( x^2 + 100y^2 \)

In problems 20 – 26, factor the following completely. (Objective: Factor Trinomials with Leading Coefficient = 1)

20. \( x^2 + 7x + 12 \)
21. \( y^2 - 12y + 27 \)
22. \( x^2 - 22 - 9x \)
23. \( x^2 + 3x - 7 \)
24. \( 16 + y^2 - 8y \)
25. \( x^4 - x^2 - 6 \)
26. \( a^4 + 5a^2 - 24 \)

In problems 27 – 36, factor the following completely. (Objective: Combine All Factoring Strategies)

27. \( 3x^2 - 192 \)
28. \( a^2 + 25 - 10a \)
29. \( x^3 - 18x^2 + 81x \)
30. \( x^3 + 3x^2 - 4x - 12 \)
31. $50y^2 - 32$
32. $x^4 + 16$
33. $x^3y + 8x^2y + 5xy$
34. $3(x - 4) - 6x^2(x - 4)$
35. $x^5 - 4x^4 + 3x^3$
36. $t^8 - 1$
37. $x^4yz^2 + 2x^3yz^2 - 4x^2yz^2 - 8xyz^2$
38. $2xy^2(x^2 - 100) - 8x(x^2 - 100)$
Solutions to Module V:

1. \(2x(x + 3)\)
2. \(5x^3(x^2 + 2)\)
3. \(4(2x^2 - x - 5)\)
4. \(16xy^2(x^5y^2 + 2x^4y - 3)\)
5. \(x(5x^4 + 10x - 8)\)
6. \(3xy^2(9xy^5 + 5x^4y - 1)\)
7. \((x + 3)(x^2 + 2)\)
8. \((4x - 3)(2x^2 - 5)\)
9. \((x + 7)(x^2 - 2)\)
10. \((6x - 7)(3x^2 + 5)\)
11. \((x - 2)(7x^2 - 1)\)
12. \((x + 6)(2x^2 - 5)\)
13. \((5x - 1)(4x^2 - 5)\)
14. \((x + 11)(x - 11)\)
15. \((x + 9y)(x - 9y)\)
16. \((x + 7)(x - 7)\)
17. \((5x + y^3)(5x - y^3)\)
18. \((3x^2 + 4y)(3x^2 - 4y)\)
19. Prime
20. \((x + 4)(x + 3)\)
21. \((x - 9)(x - 3)\)
22. \((x - 11)(x + 2)\)
23. Prime
24. \((y - 4)^2\)
25. \((x^2 - 3)(x^2 + 2)\)
26. \((a^2 + 8)(a^2 - 3)\)
27. \(3(x + 8)(x - 8)\)
28. \((a - 5)^2\)
29. \(x(x - 9)^2\)
30. \((x + 3)(x + 2)(x - 2)\)
31. \(2(5x + 4)(5x - 4)\)
32. Prime
33. \(xy(x^2 + 8x + 5)\)
34. \(3(x - 4)(1 - 2x^2)\)
35. \(x^3(x - 3)(x - 1)\)
36. \((t^4 + 1)(t^2 + 1)(t + 1)(t - 1)\)
37. \(xyz^2(x + 2)^2(x - 2)\)
38. \(2x(x + 10)(x - 10)(y + 2)(y - 2)\)
Module VI
Linear Equations, Formulas, and Inequalities

In Modules IV and V we learned how to create, evaluate, simplify, and factor polynomials that help us problem solve. In this module we will focus on solving equations, formulas, and inequalities. We will begin by learning how to identify and solve first-degree (linear) equations in one variable. Once we have sharpened our equation solving skills, we will take a look at literal equations or formulas (equations with many variables), and then solve for a given variable. We will then use this knowledge to apply formulae of finance. Finally, we will introduce inequalities and ways to express infinite solutions.

I. Solving First-Degree (Linear) Equations

Khan Academy Resources: https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-1

A first-degree or linear equation is an equation that is made up of monomials or polynomials that are of at most degree 1. A solution to a linear equation is any number that when replaced for the variable in the equation, creates a true statement, that is, a solution is any value for the variable that satisfies the equation. We can check solutions by substituting our answer into the original equation, otherwise, if we made a mistake in the altered equation, our substitution may not reflect the error.

Tools for Solving First Degree (Linear) Equations:
Goal: Get “x” (or the variable) alone on one side of the equation and everything else on the other side.

1. Simplify both sides of the equation:
   a. Clear parentheses by using the distributive property.
   b. Clear fractions by multiplying the entire equation by the LCD (Lowest Common Denominator).
   c. Clear decimals by multiplying the entire equation by an appropriate power of 10. We choose the power of 10 by how many decimal places we want to move to get a whole number.
   d. Combine like terms, making sure to work with each side of the equation separately.

2. Use the addition/subtraction principle. We can add or subtract a constant to or from both sides of the equation without affecting the solution of the equation. For example:
   a. Consider the equation $x - 5 = 7$. To isolate “x”, we must add 5 on both sides, giving us $x - 5 + 5 = 7 + 5$, or $x = 12$.
   b. Consider the equation $x + 5 = 7$. To isolate “x”, we must subtract 5 on both sides, giving us $x + 5 - 5 = 7 - 5$, or $x = 2$.

3. Use the multiplication/division principle. We can multiply or divide both sides of the equation by a constant without affecting the solution of the equation. For example:
a. Consider the equation $5x = 20$. To isolate “$x$”, we must divide both sides by $5$, giving us $\frac{5x}{5} = \frac{20}{5}$, or $x = 4$.

b. Consider the equation $\frac{x}{5} = 20$. To isolate “$x$”, we must multiply both sides by $5$, giving us $5\left(\frac{x}{5}\right) = 20$, or $\frac{5x}{5} = 100$, or $x = 100$.

4. Check your answer by substituting the answer back into the original equation.

**Example 1:** Solve the following first degree equations for $x$ and check your answer:

a) $5x – 10 + x = 7x + 18 – 5x$

b) $\frac{5}{16}x + \frac{3}{8}x = 2 + \frac{1}{4}x$

c) $19 – (2x + 3) = 2(x + 3) + x$

d) $6.8x + 4.6 = 10.72$

**Solutions to Example 1:**

a) First, simplify both sides of the equation by collecting like terms: $5x – 10 + x = 7x + 18 – 5x$ becomes $6x – 10 = 2x + 18$. Next, use the addition/subtraction principle and subtract $2x$ from both sides of the equation to get: $4x – 10 = 18$. Next, use the addition/subtraction principle and add $10$ to both sides of the equation to get: $4x = 28$. Finally, use the multiplication/division principle and divide both sides by $4$ to get $x = 7$. It would be good to check your answer here by substituting $7$ into the equation. Does $5(7) – 10 + 7 = 7(7) + 18 – 5(7)$? $32 = 32$, so our answer is correct.

**Check:** $5(7) – 10 + 7 = 7(7) + 18 – 5(7); \ 35 – 10 + 7 = 49 + 18 – 35; \ 32 = 32$. It works.

**Answer:** $x = 7$

b) Since fractions are involved in this equation, it would be helpful to eliminate them by multiplying the entire equation by the lowest common denominator (LCD) of all of the fractions in the equation. In this case the LCD is $16$. $16\left(\frac{5}{16}x + \frac{3}{8}x = 2 + \frac{1}{4}x\right) \Rightarrow 5x + 6x = 32 + 4x$. The equation is much more manageable now. Combining like terms we get $11x = 32 + 4x$. Subtracting $4x$ from both sides and then dividing by $7$ we get $x = \frac{32}{7}$. Check the answer to see that it satisfies the equation.

**Check:** $\frac{5}{16}\left(\frac{32}{7}\right) + \frac{3}{8}\left(\frac{32}{7}\right) = 2 + \frac{1}{4}\left(\frac{32}{7}\right), \ \frac{10}{7} + \frac{12}{7} = 2 + \frac{8}{7}, \ \frac{22}{7} = \frac{22}{7}$. It works.

**Answer:** $x = \frac{32}{7}$
c) Since this equation has parentheses, we will first distribute to clear them. $19 - (2x + 3) = 2(x + 3) + x$ becomes: $19 - 2x - 3 = 2x + 6 + x$. Now we can combine like terms on both sides to get: $16 - 2x = 3x + 6$. Adding $2x$ on both sides and subtracting $6$ on both sides we get: $10 = 5x$. Finally, divide both sides by $5$ to get $2 = x$, which means $x = 2$. Check the answer to see that it satisfies the equation. **Check:** $19 - (2(2) + 3) = 2((2) + 3) + (2), \ 19 - (4 + 3) = 2(5) + 2, \ 19 - 7 = 10 + 2, \ 12 = 12$. It works. **Answer:** $x = 2$

d) Since this equation has decimals, we will first multiply the entire equation by an appropriate power of $10$ to eliminate them. We have to move the decimal point in $10.72$ two places, so we will choose $100. \ 100(6.8x + 4.6 = 10.72)$ turns into $680x + 460 = 1072$ (this step is optional). Subtracting $460$ on both sides we get: $680x = 612$. Finally, divide both sides by $680$ to get $x = 0.9$. Check the answer to see that it satisfies the equation. **Check:** $19 - (2(2) + 3) = 2((2) + 3) + (2), \ 19 - (4 + 3) = 2(5) + 2, \ 19 - 7 = 10 + 2, \ 12 = 12$. It works. **Answer:** $x = 2$

II. Literal Equations and Formulas

Khan Academy Resources: [https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/solving-for-a-variable](https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/solving-for-a-variable)

A literal equation or formula is an equation that has multiple variables. When creating mathematical models it is often helpful to rewrite formulas in terms of different variables depending on what information is given. For instance, if we are asked to find the area of a circle given the radius, we would simply use the formula $A = \pi r^2$ and substitute the value of $r$ into the formula, then simplify. Now suppose that we wanted to create a circle that has a given area. To do this we would need to determine the circles radius. We could certainly use the formula $A = \pi r^2$ and substitute the given value $A$ into the formula and find $r$, but an easier way would be to rewrite the formula as $r = \sqrt{\frac{A}{\pi}}$.

This process is helpful if we are performing repeated computations and especially if we are using a computer spreadsheet (such as Excel) in which we can input formulas so that the computer does all the work.

Solving Literal Equations or Formulas for a Variable:
To solve a literal equation or formula for a variable, we use the same process used for solving first-degree equations. Refer back to section I for those rules. In short, we can isolate the variable we are asked to solve for by using the addition/subtraction principle or the multiplication/division principle.

**Example 2:** Solve each of the given formulas for the indicated variable.

a) $A = \frac{a + b + c}{3}$, \ for $c$ (used for finding the average of three given values)

b) $F = \frac{9}{5}C + 32$, \ for $C$ (used for converting degrees Celsius to degrees Fahrenheit)
Solution to Example 2:
a) First we will multiply the equation by the LCD (3) to eliminate the fraction:
\[ 3 \left( A = \frac{a + b + c}{3} \right) \Rightarrow 3A = a + b + c. \] Since we are solving for c, we will subtract a and b from both sides to isolate c: 
\[ 3A = a + b + c \Rightarrow 3A - a - b = c. \] Since A and a are NOT like terms, we cannot simplify the equation any further. **Answer:** \( c = 3A - a - b \)

b) Multiply the entire equation by the LCD (5) to eliminate the fraction:
\[ 5 \left( F = \frac{9}{5}C + 32 \right) \Rightarrow 5F = 9C + 160. \] Subtract 160 and divide by 9 to get:
\[ \frac{5F - 160}{9} = C. \] **Answer:** \( C = \frac{5F - 160}{9} \)

### III. Formulas to Calculate Interest, Savings & Loan Payments


The formulas that we introduce in this section will prove to be useful at some point in our lives. Because many of us require loans to make large purchases or invest money in a bank, whether the investment is in one lump sum, or on a regular payment schedule (such as a retirement fund), we would like to determine the interest that we are paying or earning. In this section we will discuss interest formulas, savings plan formulas, and loan payment formulas. We will begin with interest formulas.

Interest can be calculated two different ways:
1. **Simple Interest** (A.K.A. Per Annum): Interest that is calculated from the principal (or initial investment) only.
2. **Compound Interest**: Interest that is calculated from the principal and the prior interest earned.

The table below is an example of how an investment of $1000 at an annual percentage rate of 5% grows in time for simple interest vs. compound interest (compounding annually):

<table>
<thead>
<tr>
<th>Time (yrs.)</th>
<th>Simple Interest (Int. earned, total amt.)</th>
<th>Compound Interest (Int. earned, total amt.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$50, $1050</td>
<td>$50, $1050</td>
</tr>
<tr>
<td>2</td>
<td>$50, $1100</td>
<td>$52.50, $1102.50</td>
</tr>
<tr>
<td>3</td>
<td>$50, $1150</td>
<td>$55.13, $1157.63</td>
</tr>
<tr>
<td>4</td>
<td>$50, $1200</td>
<td>$57.88, $1215.51</td>
</tr>
<tr>
<td>5</td>
<td>$50, $1250</td>
<td>$60.77, $1276.28</td>
</tr>
</tbody>
</table>
Formulas for Simple Interest:

1. \( I = Prt \)
   - \( I \) = Interest
   - \( P \) = Principal (or initial investment)
   - \( r \) = annual interest rate (APR) in decimal form
   - \( t \) = time (in years)
2. \( A = P(1 + rt) \)
   - \( A \) = Account balance (A.K.A. “Future Value”)
   - \( P \) = Principal
   - \( r \) = annual interest rate (APR in decimal form)
   - \( t \) = time (in years)

Formulas for Compound Interest:

1. For \( n \) number of compounding per year: \( A = P \left(1 + \frac{r}{n}\right)^{nt} \)
   - \( A \) = balance in the account after \( t \) years (A.K.A. “Future Value”)
   - \( P \) = principal (or amount invested)
   - \( r \) = annual interest rate (APR in decimal form)
   - \( n \) = number of times the account is compounded in a year
2. For continuous compounding: \( A = Pe^{rt} \)
   - \( A \) = balance in the account after \( t \) years
   - \( P \) = principal (or amount invested)
   - \( r \) = annual interest rate (APR in decimal form)
   - \( e \) is an irrational number and is approximately equal to 2.718

Note: For interest compounded daily, banks often use \( n = 360 \) rather than \( n = 365 \).

How do we know when to use a simple interest formula or a compound interest formula? If we are computing compound interest, the problem will include the word “compounded”, otherwise we can assume that the interest is simple.

Example 3: For each of the following, find the balance in an account after the given term.
(assume no deposits or withdrawals are made during the entire term.)
a) $20000 invested in an account paying 3% simple interest for 20 years.
b) $20000 invested in an account paying 3% interest compounded quarterly for 20 years.
c) $20000 invested in an account paying 3% interest compounded continuously for 20 years.
Solutions to Example 3:

a) In this example we will use the formula $A = P(1 + rt)$ since we are calculating the account balance for simple interest. Substituting $P = 20000$, $r = 0.03$, and $t = 20$ we get:

$$A = 20000(1 + (0.03)(20)) = 32000.$$ Answer: $32,000$

b) In this example we will use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ because we are calculating compound interest, and it is not continuous. Substituting $P = 20000$, $r = 0.03$, $n = 4$ (quarterly means 4 times in a year), and $t = 20$ we get:

$$A \approx 36360.88.$$ Answer: $36,360.88$

c) In this example we will use the formula $A = Pe^{rt}$ since we are calculating interest compounded continuously. Substituting $P = 20000$, $r = 0.03$, and $t = 20$ we get:

$$A \approx 36,442.38.$$ Answer: $36,442.38$

Savings Plan:

In most cases, one doesn’t have the luxury of being able to put a large sum of money into the bank and watch it grow exponentially over time. For most of us, setting up a savings plan in which we deposit a small sum of money, say, on a monthly basis is a more realistic option. Now we will be discussing formulas that help us find the future value of such savings plans.

Savings Plan Formula:

$$A = \frac{P\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{N}}$$

- $A =$ Accumulated Balance
- $P =$ Regular deposits or payment amount
- $r =$ interest rate (APR) in decimal form
- $n =$ number of times the account is compounded in a year
- $N =$ number of payment periods per year
- $t =$ time in years

NOTE: For our examples with savings plans we will assume that the number of times the account is compounded in a year, $n$, is the same as the number of payment periods per year, $N$.

Example 4: At age 25 you wish to set up a retirement account so that when you retire at the age of 65 you’ll have saved $750,000. If your investment plan pays an APR of 6.5%, how much should your monthly deposit be?
Solution to Example 4: 
Substituting \( A = 750000 \) (we want 750000 in the account), \( r = 0.065 \) (APR in decimal form), \( n = 12 \) (since we are making payments every month, there are 12 months in a year), and \( t = 40 \) (since we want to retire in 65 – 25 years) into the savings plan formula we get:

\[
P = \frac{750000}{\frac{0.065}{12} \left(1 + \frac{0.065}{12}\right)^{12 \cdot 40} - 1}.
\]

Solving for \( P \) we get:

\[
\frac{750000}{\left(1 + \frac{0.065}{12}\right)^{12 \cdot 40} - 1} = P.
\]

Being very careful with our calculator we get \( P \) is approximately $328.43. Answer: $328.43

Suppose that we would like to purchase a home or a car. In most cases one would not have enough money to make the purchase without taking out a loan. We will now learn how to calculate payments on installment loans, that is, a loan that you pay off with equal regular payments (A.K.A. amortized loan).

Since regular payments are being made we need to consider our savings plan formula and since compound interest is calculated on the loan we need to also consider our compound interest formula. Setting these two formulas equal to one another and solving for \( P \) in the savings plan formula we can obtain the installment loan payment formula.

**Installment Loan Payment Formula:**

\[
PMT = \frac{P \cdot \frac{r}{n}}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}
\]

- \( P = \) Loan Principal
- \( r = \) interest rate (APR) in decimal form
- \( n = \) number of payment periods per year
- \( t = \) loan term in years

Note: For installment loans early in the loan term, the portion going toward interest is relatively high and the portion going toward the principal is relatively low. As the term proceeds, the portion going toward interest gradually decreases and the portion going toward principal gradually increases.
Terms Used In Loans:

**Mortgage**: Installment loan to buy a home. We will consider fixed rate home mortgages.

**Down Payment**: Money paid toward the initial cost of the home.

**Closing Costs**: Fees at the time you take out a loan.

**Points**: A fee a lender charges. 1 point is 1% of the loan amount. Many lenders divide points into two categories: an “origination fee” that is charged on all loans and “discount points” that vary for loans with different rates. One might purchase a point to buy down the interest rate on a loan.

In our examples, we will ignore down payment, closing costs, and points.

**Example 5**: A home mortgage of $200,000 with a fixed APR of 7.5% for 30 years is borrowed. Calculate the monthly payment.

Solution to Example 5:
Substituting $P = 200000$, $r = 0.075$, $n = 12$ (monthly payments, 12 in a year), and $t = 30$ into the loan payment formula we get:

$$PMT = \frac{200000 \cdot 0.075}{12 \left(1 - \left(1 + \frac{0.075}{12}\right)^{-12 \cdot 30}\right)} \approx 1398.43.$$  

Answer: $1,398.43$

The answer given in Example 8 is lower than what an actual mortgage lender would quote for this loan, since property taxes and insurance has not been figured in to the calculation. Those amounts vary on location.

**SEE ACCOUNTING PROBLEMS TO GET PRACTICE WITH THIS SECTION**

**IV. Problem Solving with Linear Equations**


When solving problems in this module it is helpful to ask ourselves the following questions before we begin:
1. Are we given a formula with the problem to use?
2. Can I identify what I am asked to find? If so, we should define our variable to be this unknown.
3. If a formula is not given, can we relate the situation to a first-degree equation? If so, can we write an equation and solve it?
**Example 6:** Suppose that to get an A- in MAT 107, you must have an average between 90 and 93 (inclusive) on five tests of 100 points each. The scores on your first four tests were 87, 92, 84, and 96. What must you score on the fifth test to get an average of 90 for the course?

Solution to Example 6:
We are not given an equation, but we do know how to calculate the average (arithmetic mean). To find the mean we need to add all of the test scores together, then divide by the number of tests considered. In this case, we have one unknown test, the fifth test. Let \( x \) = the score on the fifth test. Knowing that the average is 90, we can write an equation:

\[
90 = \frac{87 + 92 + 84 + 96 + x}{5}.
\]

Simplifying the equation, we get:

\[
90 = \frac{359 + x}{5}.
\]

Solving the equation we get \( x = 91 \). **Answer:** We must score 91 on the fifth test to get an average of 90.

**Break-Even Analysis:**
When owning a business it is important to know what volume of sales will produce a profit. When a business breaks even, then the business had earned a profit of 0. Another way to compute the break-even point would be to set the businesses revenue expression equal to the businesses cost expression, that is, in order for a business to break-even, the business must earn enough revenue to offset the costs.

**Break-Even Point:** Revenue = Costs

**Example 7:** Given a restaurant sales of $180,000, variable costs of $72,000 and fixed costs of $95,000. Assuming the variable cost %, and the fixed costs holds constant, what is the break-even Point?

Solution to Example 7:
Since we are asked to find the break-even point, and we are not given formulas for revenue, or costs, we need to develop expressions for the revenue and costs. Since the variable cost percentage is constant and the fact that the variable costs ($72,000) is 40% of the total sales ($180,000) we can begin to write an equation. First, let \( x \) = amount of total sales. This will be our revenue expression. Next, if \( x \) is our total sales, then the cost expression is 0.4\( x \) + 95000 (variable costs + fixed costs). Now we will set the revenue expression equal to the cost expression to obtain the equation: \( x = 0.4x + 95000 \). Solving for \( x \) we get \( x = 158,333.33 \). **Answer:** $153,333.33 in sales to break-even
V. The Language of Linear Inequalities

Suppose that on a business trip you are given a budget of $250 for the rental car. The rental car company charges $45 per day plus $0.10 per mile. If you need a rental car for two days, how many miles can you drive to stay within your budget? This problem translates to an inequality: “The total rental car charge must be less than or equal to $250.”

Many times when a situation translates to an equation that has an interval of possible solutions, or an infinite number of possible solutions, then we use inequalities in place of the equal sign. If the inequality is linear (all terms have at most degree 1), then it is a linear inequality. We first used inequalities in Module I when ordering real numbers. We will extend the concept here beginning with defining terms used with inequalities.

Terms Used With Inequalities:

**Inequality:** An inequality is any sentence containing <, >, ≤, ≥, or ≠.
- a) < translates to “less than”.
- b) > translates to “greater than”.
- c) ≤ translates to “less than or equal to”.
- d) ≥ translates to “greater than or equal to”.
- e) ≠ translates to “is not equal to”. If we say that $x \neq 3$, we can conclude that $x < 3$ or $x > 3$.

**Solution to an Inequality:** Any value for the variable that makes an inequality true is called a solution of the inequality.

**Solution Set:** The set of all possible solutions to an inequality (or equation).

**Interval Notation:** A convenient way to write all of the solutions of an inequality in one variable. Interval notation uses parentheses, ( ), and brackets, [ ] to describe open intervals and closed intervals.

**Open Interval:** The open interval $(a, b)$ means $a < x < b$. This means $x$ CANNOT equal $a$ or $b$, but $x$ can be all the values in between $a$ and $b$.

**Closed Interval:** The closed interval $[a, b]$ means $a \leq x \leq b$. This means that $x$ CAN equal $a$ or $b$ and all the values in between $a$ and $b$.

**Half-Open Interval:** The half-open interval $(a, b]$ means $a < x \leq b$, and the half-open interval $[a, b)$ means $a \leq x < b$.

**Infinity and Negative Infinity** ($\infty$ and $-\infty$): Since interval notation describes an interval of solutions, it is important that we define the boundary values or endpoints. $\infty$ or $-\infty$ are used in intervals.
give an endpoint to an interval that has no end. For instance the set of all real numbers in interval notation is given to be \((-\infty, \infty)\) since there is no end (in either direction) of all possible real numbers. Infinity and negative infinity are NOT numbers, they are used in a way (mathematically) to describe something that doesn’t have a bound.

**Set-Builder Notation:** Another way to write all of the solutions of an inequality in one variable. Set-builder notation is better utilized with systems of linear equations in two variables; however, it may be used with inequalities. For example, to write the solution \(x > 5\) in set-builder notation, we would write: \(\{x \mid x > 5\}\), which translates to “The set containing all \(x\)-values such that \(x\) is greater than 5.” Notice that the vertical line, “\(\mid\)”, is used to mean “such that”. Don’t confuse this with the absolute value symbol.

### Translating Inequalities, Set-Builder Notation, and Interval Notation:

<table>
<thead>
<tr>
<th>Word Form</th>
<th>Translation</th>
<th>Set-Builder Notation</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x) is greater than (b)</td>
<td>(x &gt; b)</td>
<td>({x \mid x &gt; b})</td>
<td>((b, \infty))</td>
</tr>
<tr>
<td>(x) is more than (b)</td>
<td>(x &gt; b)</td>
<td>({x \mid x &gt; b})</td>
<td>((b, \infty))</td>
</tr>
<tr>
<td>(x) must exceed (b)</td>
<td>(x &gt; b)</td>
<td>({x \mid x &gt; b})</td>
<td>((b, \infty))</td>
</tr>
<tr>
<td>(x) is less than (b)</td>
<td>(x &lt; b)</td>
<td>({x \mid x &lt; b})</td>
<td>((-\infty, b))</td>
</tr>
<tr>
<td>(x) is exceeded by (b)</td>
<td>(x &lt; b)</td>
<td>({x \mid x &lt; b})</td>
<td>((-\infty, b))</td>
</tr>
<tr>
<td>(x) is greater than or equal to (b)</td>
<td>(x \geq b)</td>
<td>({x \mid x \geq b})</td>
<td>([b, \infty))</td>
</tr>
<tr>
<td>(x) is at least (b)</td>
<td>(x \geq b)</td>
<td>({x \mid x \geq b})</td>
<td>([b, \infty))</td>
</tr>
<tr>
<td>(x) is no less than (b)</td>
<td>(x \geq b)</td>
<td>({x \mid x \geq b})</td>
<td>([b, \infty))</td>
</tr>
<tr>
<td>(x) is less than or equal to (b)</td>
<td>(x \leq b)</td>
<td>({x \mid x \leq b})</td>
<td>((-\infty, b])</td>
</tr>
<tr>
<td>(x) is at most (b)</td>
<td>(x \leq b)</td>
<td>({x \mid x \leq b})</td>
<td>((-\infty, b])</td>
</tr>
<tr>
<td>(x) cannot exceed (b)</td>
<td>(x \leq b)</td>
<td>({x \mid x \leq b})</td>
<td>((-\infty, b])</td>
</tr>
<tr>
<td>(x) is no more than (b)</td>
<td>(x \leq b)</td>
<td>({x \mid x \leq b})</td>
<td>((-\infty, b])</td>
</tr>
</tbody>
</table>

Note: Inequalities can be read from right to left or left to right. The inequality \(2 \geq x\) can be read as “2 is greater than or equal to \(x\)” or “\(x\) is less than or equal to 2”. That is, \(2 \geq x\) means \(x \leq 2\).

**Example 8:** Translate each sentence to an algebraic sentence involving “\(x\)” with use of an inequality. Then write the inequality in set-builder notation and interval notation.

a) An elevator can hold no more than 2000 lbs.

b) You must spend more than $50 to receive the discount.
Solution to Example 8:
a) First determine the value “x”. In this case we will let x = Elevator’s weight capacity. Next, determine the inequality that we will use. “no more” translates to less than or equal to. Writing the inequality we get: \( x \leq 2000 \). **Answer: x \leq 2000**

In **set-builder notation** we write: \( \{ x \mid x \leq 2000 \} \)

In **interval notation** we write: \( (-\infty, 2000] \)

b) First determine the value “x”. In this case we will let x = Amount needed to spend to get discount. Next, determine the inequality that we will use. “more than” translates to greater than. Writing the inequality we get: \( x > 50 \). **Answer: x > 50**

In **set-builder notation** we write: \( \{ x \mid x > 50 \} \)

In **interval notation** we write: \( (50, \infty) \)

V. Graphing Linear Inequalities


We’ve just learned how to express solutions to linear inequalities algebraically using set-builder notation or interval notation. Another way to express solutions of inequalities would be graphically. Since the solution to the types of linear inequalities we are solving involve just one variable, we will graph the solution on the **number line** such as the one given below.

```
-5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5
```

**Tools for Graphing Linear Inequalities:**

1. Order the inequality, that is, “x” on the left and “number” on the right.
2. Determine the **boundary value** or **endpoint** to be placed on the number line.
3. Determine whether you will use an “open dot” or a “solid dot” then place the dot where the boundary number is. **The dot is open for > or <. The dot is solid for \( \geq \) or \( \leq \).** In some textbooks, parentheses ( ) are used in place of open dots and brackets [ ] are used in place of solid dots. We will use open and solid dots here.
4. Determine the **direction of the line** from the inequality then draw the line.
Example 9: Graph each inequality on a number line and write the solution set in interval notation:

a) \( x > 3 \)

b) \( x \leq 2 \)

c) \( -1 > x \)

Solution to Example 9:

a) The boundary value is 3. The inequality is >, so we will place an open dot at 3 on the number line. The direction of the line for all numbers greater than 3 would be a line to the right of 3.

Answer:

Interval notation: \((3, \infty)\)

b) The boundary value is 2. The inequality is \(\leq\), so we will place a solid dot at 2 on the number line. The direction of the line for all numbers less than or equal to 2 would be a line to the left of 2.

Answer:

Interval Notation: \((-\infty, 2]\)

c) In this case we should order the inequality: \(-1 > x\) means \(x < -1\). The boundary value is -1. The inequality is <, so we will place an open dot at -1 on the number line. The direction of the line for all numbers less than -1 would be a line to the left of -1.

Answer:

Interval Notation: \((-\infty, -1]\)
V. Solving Linear Inequalities


We solve linear inequalities in the same way that we solve linear equations, that is, we can use the addition/subtraction principle and the multiplication/division principle in the same way that we use them with linear equations EXCEPT when we multiply or divide by a negative number. **If we multiply or divide the inequality by a negative number we must “flip” the inequality.** For example if we wanted to solve \(-2x < 6\), we would divide both sides by “\(-2\)”, but since we’re dividing by a negative number we must “flip” the inequality. The solution would then be \(x > -3\).

Why do we “flip” the inequality when we multiply or divide by a negative number? Suppose we wanted to solve the linear inequality \(-2x > 6\) WITHOUT dividing by “\(-2\)”. The steps we would take would look something like this:

\[
\begin{align*}
-2x &> 6; \text{ add } 2x \text{ to both sides} \quad -2x + 2x > 6 + 2x \quad \text{to get } 0 > 6 + 2x. \\
\text{Subtract } 6 \text{ from both sides} &\quad 0 - 6 > 6 + 2x - 6 \quad \text{to get } -6 > 2x \\
\text{Divide both sides by } 2 &\quad \frac{-6}{2} > \frac{2x}{2} \quad \text{or } -3 > x
\end{align*}
\]

We get \(-3 > x\), which means \(x < -3\). It would be much easier for us to take \(-2x < 6\) and divide both sides by “\(-2\)” and “flip” the inequality to get \(x > -3\).

**Example 10:** Solve the following linear inequalities and write your answer in interval notation:

a) \(-3x + 7 < -2\)
b) \(5x + 6 ≥ 3x - 2\)
c) \(1 - (2x + 8) < (x - 9) - 4x\)

**Solution to Example 10:**

a) 
\(-3x + 7 < -2\), subtract 7 on both sides, 
\(-3x < -9\), divide both sides by \(-3\) and flip the inequality, 
\(x > 3\).

Write the answer in interval notation: \((3, \infty)\)

**Answer:** \((3, \infty)\)
b) 
\[ 5x + 6 \geq 3x - 2, \text{ subtract } 3x \text{ on both sides,} \]
\[ 2x + 6 \geq -2, \text{ subtract } 6 \text{ on both sides,} \]
\[ 2x \geq -8, \text{ divide both sides by } 2, \]
\[ x \geq -4 \]
Write the answer in interval notation: \([-4, \infty)\)
**Answer:** \([-4, \infty)\)

c) 
\[ 1 - (2x + 8) < (x - 9) - 4x, \text{ distribute to clear parentheses,} \]
\[ 1 - 2x - 8 < x - 9 - 4x, \text{ combine like terms,} \]
\[ -2x - 7 < -3x - 9, \text{ add } 3x \text{ to both sides,} \]
\[ x - 7 < -9, \text{ add } 7 \text{ to both sides,} \]
\[ x < -2 \]
Write the answer in interval notation: \((-\infty, -2)\)
**Answer:** \((-\infty, -2)\)

**V. Compound Inequalities: An Introduction**


In this section we will introduce compound inequalities. Specifically, we will focus on the definition of a compound inequality, graphing them on a number line, and writing them in interval notation.

**Compound Inequalities:**
When two or more inequalities are connected with the word “and” or the word “or” then the combination is called a **compound inequality**. A conjunction uses the word “AND” to connect the inequalities, and a disjunction uses the word “OR” to connect the inequalities.

1. **Conjunction (“AND”):** The solution to a conjunction is the **intersection** between the two inequalities. That is, the solution is the numerical values that BOTH inequalities have in common. The mathematical symbol for an intersection of two intervals is \(\cap\). By writing \(a < x \text{ AND } x < b\), we can conveniently write this compound inequality without the word “AND” as \(a < x < b\), which translates to “\(x\) is between \(a\) and \(b\)”.

**Example 11:** Use inequalities to translate the situation to an algebraic sentence: “My target weight loss is between 20 and 30 lbs.”
Solution to Example 11: Let \( x = \) my target weight loss. By saying the target weight loss is between 20 and 30 lbs, we can write \( 20 < x < 30 \).
Answer: \( 20 < x < 30 \)

2. **Disjunction** ("OR"): The solution to a disjunction is any numerical value that satisfies EITHER of the inequalities. This solution corresponds to the union of the two inequalities, and the mathematical symbol for union is \( \cup \). There is no convenient way to write a disjunction without using the word "OR" as there was with a conjunction.

**Translating Compound Inequalities to Interval Notation:**

<table>
<thead>
<tr>
<th>Compound Inequality Conjunctions</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assume a is less than b</td>
<td></td>
</tr>
<tr>
<td>( a &lt; x &lt; b )</td>
<td>((a, b))</td>
</tr>
<tr>
<td>( a \leq x &lt; b )</td>
<td>([a, b))</td>
</tr>
<tr>
<td>( a &lt; x \leq b )</td>
<td>((a, b])</td>
</tr>
<tr>
<td>( a \leq x \leq b )</td>
<td>([a, b])</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compound Inequality Disjunctions</th>
<th>Interval Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assume a is less than b</td>
<td></td>
</tr>
<tr>
<td>( x &lt; a \text{ or } x &gt; b )</td>
<td>((-\infty, a) \cup (b, \infty))</td>
</tr>
<tr>
<td>( x &lt; a \text{ or } x \geq b )</td>
<td>((-\infty, a) \cup [b, \infty))</td>
</tr>
<tr>
<td>( x \leq a \text{ or } x &gt; b )</td>
<td>((-\infty, a] \cup (b, \infty))</td>
</tr>
<tr>
<td>( x \leq a \text{ or } x \geq b )</td>
<td>((-\infty, a] \cup [b, \infty))</td>
</tr>
</tbody>
</table>

**Tools for Graphing Linear Inequalities:**
1. Determine if the compound inequality is a conjunction or disjunction (AND or OR).
2. Determine the **boundary values** or endpoints to be placed on the number line.
3. For each boundary value, determine whether you will use an “open dot” or a “solid dot” then place the dot where the boundary number is. **The dot is open for > or <. The dot is solid for ≥ or ≤.**
4. Determine the **direction of the line or lines** from the inequality then draw them. For the conjunctions that we are studying here the line should always connect in the middle of the boundary values, and for disjunctions the lines should always go in opposite direction.
Example 12: Graph the following on a number line and write in interval notation:

a) $-2 < x \leq 4$

b) $x \leq -1$ or $x > 3$

a) This is a conjunction (AND). The boundary values are -2 and 4. The inequality is $<$ next to -2, so we will place an open dot at -2 on the number line. The inequality is $\leq$ next to 4 so we will place a solid dot at 4 on the number line. The conjunction suggests our solutions are everything between the boundary values -2 and 4, so we will connect the dots with a line in between.

Answer:

Interval notation: $(-2, 4]$  

b) This is a disjunction (OR). The boundary values are -1 and 3. The inequality is $\leq$ next to -1, so we will place a solid dot at -1 on the number line. The inequality is $>$ next to 3 so we will place an open dot at 3 on the number line. The disjunction suggests our solutions are everything less than or equal to the boundary value -1 and everything greater than the boundary value 4, so we will draw lines in opposite direction.

Answer:

Interval notation: $(-\infty, -1] \cup (3, \infty)$

Homework Set:

In problems 1 – 15, solve the following first-degree (linear) equations for $x$. Check your solutions.

1. $6x - 5 = 2$
2. $6x - x + 4 = -x - 7 + 5$
3. $4x - 2(x + 6) = 5x - 3$
4. $6(x - 2) - 2 = 3x - 5$
5. $4x - 3x + 2 = 5x + 7$
6. \(7 - 3(x + 4) = 5x - (x - 9)\)
7. \(3x - [4(x - 3) + 10] = -2(x + 4) - 15\)
8. \(\frac{x}{3} + 1 = \frac{1}{2}x + 7\)
9. \(\frac{2x - 4}{3} + \frac{x + 2}{2} = \frac{x - 1}{6}\)
10. \(0.4x - 0.25 + 2x = 0.01x - 0.3\)
11. \(0.4x + 3.8 = 1.6x + 1.4 - 0.6x\)
12. \(4x + 2 - 5x = 7 - 6x\)
13. \(-2x + 7(x + 2) = 14\)
14. \(\frac{2}{3}(12x + 5) = 10\)
15. \(\frac{3}{2}(x + 5) - \frac{1}{4}(x + 24) = 0\)

In problems 16 – 24, solve the following literal equations for the indicated variable.

16. Solve for \(r\): \(A = P + Prt\)
17. Solve for \(P\): \(A = P(1 + rt)\)
18. Solve for \(r\): \(S = \frac{a}{1 - r}\)
19. Solve for \(x\): \(x + 5 = 3x - a\)
20. Solve for \(x\): \(Ax + By + C = 0\)
21. Solve for \(a\): \(3a + b = 8a + c\)
22. Solve for \(p\): \(\frac{mn}{2pr} = 20\)
23. Solve for \(n\): \(m = \frac{3n + 2k}{7k}\)
24. Solve for \(v\): \(x = s + vt\)

In problems 25 – 30, translate to a linear equation, then solve.

25. One number is four more than a second number. The sum of the numbers is thirty-four. Find the numbers.
26. The perimeter for a rectangular building is 300 feet. If the length is four times the width, what will be the dimensions of the building?
27. A 72 inch piece of plywood is cut into two pieces. One piece is two inches longer than the other. Find the lengths of the pieces.
28. The sum of three consecutive integers is 66. Find the integers.
29. The sum of three consecutive odd integers is 117. Find the integers.
30. Funky Grill Rent-A-Car rents cars at a daily rate of $34.50 plus $0.55 per mile. Brooklyn rents a car to deliver pizzas to her customers. If she has a daily budget of $150 for the rental car, how many miles can she drive in one day to stay within budget?
In problems 31 – 34, solve the linear inequality and write your answer in interval notation.

31. $3x + 1 \geq 2$
32. $6 - 4x \geq -8 + 3x$
33. $-\frac{1}{4}x < \frac{5}{12}$
34. $5 - (x + 2) > 5x - 4$

In problems 35 – 38, graph the inequalities on a number line and write in interval notation.

35. $x > -2$
36. $x \leq 0$
37. $x \leq -3$ or $x \geq \frac{1}{2}$
38. $-4 < x < 4$

1. APPLICATIONS FOR EMT/MEDICAL ASSISTANT/NURSING

FUN FACT: Calculating medications based on body surface area (BSA) is sometimes used for pediatric or fragile adult patients and those taking certain medications. These types of patients are sometimes at high risk and require a more precise method of medication calculation.

In problems 39 – 41, use Mosteller’s Formula to calculate the body surface area (BSA) of each patient. Round each answer to the nearest hundredth.

$$BSA = \sqrt{\frac{h \cdot w}{3600}} \text{ m}^2 \text{ where } h = \text{ height in cm and } w = \text{ weight in kg.}$$

39. Patient weight: 45.35 kg. Patient height: 142.24 cm
40. Patient weight: 90.7 kg. Patient height: 160 cm
41. Patient weight: 175 lb. Patient height: 73 inches

In problems 42 – 43, Use the half-life formula $y = a(0.5)^{t/6}$, where $a$ = the initial amount of drug, $t$ = time that the drug is left in the bloodstream (in hours), and $y$ = amount of the drug left in the bloodstream. Round each answer to the nearest tenth.

42. How much drug is left in the bloodstream 17 hours after a 280 milligram dose?
43. How much drug is left in the bloodstream 4 hours after a 500 milligram dose?
2. APPLICATIONS FOR FIRE SCIENCE

FUN FACT: Firefighters need to consider water pressure when fighting fires. To achieve a desired nozzle pressure (DNP), a few factors must be considered. First, you must note the head loss (HL) or head gain (HG). Water head is the height of the water column (lift) due to imposing pressure. The head pressure is positive (gain) if the hose lay is downhill because the force of gravity is helping push the water down, consequently increasing the pressure. The head pressure is negative (loss) if the hose lay is uphill, since the force of gravity is pulling the water down, when it needs to be pumped up. Figure 6.1 below indicates that 1 foot of water head or lift produces 0.5 pounds per square inch of pressure. On that same note, 1 pound per square inch can produce 2 feet of water head. For every foot uphill or downhill, there is a change of 0.5 pounds per square inch of pressure. Note that this measurement represents the height of the hose (elevation) and not the length of the hose.

![Figure 6.1: Water Pressure vs Height](image)

In Table 6.1 the drawings correspond to the pressure on a square inch cross section caused by the height of water above it. Note that as the column's height doubles, so does the pressure. Both exact and rounded field application values are given.

The second consideration for pump pressure calculations involves friction loss (FL). As a field rule, the pressure in a line is reduced by 5 pounds per square inch for each appliance added to the line. For example, a hose lay with five wye valves will result in a 25 pounds per square inch pressure loss due to the friction introduced by these fittings. This approximation is used to simplify calculations and is not precisely what occurs in the field.

The following formula will help a firefighter calculate the desired nozzle pressure when in the field.
Calculating Desired Nozzle and Pump Pressures:

\[ n = \text{Desired Nozzle Pressure} \]
\[ p = \text{Engine (Pump) Pressure} \]
\[ g = \text{Head Gain} \]
\[ L = \text{Head Loss} \]
\[ f = \text{Friction Loss} \]

Desired Nozzle Pressure = Engine (Pump) Pressure ± (Head Gain or Head Loss) - Friction Loss

\[ n = p \pm (g \text{ OR } L) - f \]

When calculating desired nozzle pressure in a *downhill* hose lay, *add* the head pressure (that is, add \( g \)). In *uphill* hose lays, *subtract* the head pressure (that is, subtract \( L \)). The calculations vary to account for the work of gravity.

The head pressure is expressed in terms of loss or gain. Because the pump and the nozzle are at opposite ends of the hose, head pressure that is positive at the pump will be negative at the nozzle and vice versa. It is crucial that the sign of the head pressure be correct. If the hose lay is uphill, the head pressure is negative, and if it is downhill, the head pressure is positive. Careful attention must be paid to the sign of the head gain or head loss term, and whether the gain or loss should be added or subtracted. See figure 6.2 below.

*Figure 6.2: Head gain and head loss depend on the nozzle's position relative to the pump.*
In problems 44 – 45, use the formula \( n = p \pm (g \text{ OR } L) – f \) to answer the following questions.

44. A progressive hose lay has six gated wye valves along the length of the trunk line resulting in 30 psi of friction loss. The nozzle outlet is 200 feet below the engine creating 100 psi of head gain. The desired nozzle pressure of the trunk line is 100 pounds per square inch. At what pressure does the engine need to perform?

45. Kevin is fighting a fire with a hose that has 0 fittings (i.e. 0 friction loss) and needs the nozzle pressure to be 100 pounds per square inch. He is 100 feet above the engine creating a head loss of 50 psi. What pump pressure does he need?

FUN FACT: **Spread distance (SD)** is the forward distance a fire spreads in a given amount of time. SD can be calculated from rate of spread (ROS) and projected time (PT). Spread distance can be calculated using the following formula.

\[
\text{Spread Distance} = \text{Rate of Spread} \times \text{Projected Time} \\
SD = \text{ROS} \times \text{PT}
\]

In problems 46 – 49, use the formula SD = ROS \times PT to answer the following questions.

46. What is the spread distance, in feet, for a fire that has a rate of spread of 6 chains per hour for a 3-hour time span?

47. The rate of a fire spread is 4 chains per hour. What will the spread distance be in 3 hours?

48. The projected time for a fire to spread 24 chains is 5 hours. What is the rate of spread?

49. How long will it take a fire moving at 3.2 chains per hour to spread a distance of 38.4 chains?

FUN FACT: Firefighters may need to calculate the force needed to move heavy objects in the field. The formula used to calculate force is: \( \text{Force} = \text{mass} \times \text{acceleration} \ (F = ma) \). In the U.S. Customary system, force is given in pounds, mass is given in slugs, and acceleration due to gravity is -32 feet per second every second. Acceleration is negative since it is a downward force. In the metric system, force is given in Newtons, mass in kilograms, and acceleration due to gravity is -10 meters per second every second.

In problems 50 – 53, use the formula \( F = ma \) to answer the questions.

50. A car with a mass of 88 slugs is placed on a hydraulic lift. In the trunk of the car are golf clubs with a mass of 1 slug and a tool box with a mass of 2 slugs. What force must be applied to lift the car?
51. Four concrete slabs each with a mass of 20 slugs are to be lifted by a crane. What force must be applied to lift the slabs?

52. A cable that can hold a maximum of 2500 pounds is to be used to lift 5 steel beams that each have a mass of 17 slugs. Will the rope hold?

53. A concrete slab has a mass of 25 slugs. The slab is being lifted by three cables, each able to support a maximum of 275 pounds. Will these cables be able to lift the slab?

FUN FACT: Percentages are useful for a number of fire science applications. One of these applications is estimating live fuel moisture. Live fuel moisture can be measured using oven drying and weighing procedures. Because this process is time-consuming and cannot be completed in the field, fire considerations are usually satisfied with a good estimate. Live fuel moisture can be estimated using the values in the Figure 6.3 below, which provides moisture percentages for fuels at different stages of vegetative development. For instance, from the figure, we observe that completely cured fuels have a live fuel moisture of less than 30%. This value is the result of subtracting the dry weight of the fuel from the total (wet) weight and dividing by the dry weight.

<table>
<thead>
<tr>
<th>Stage of vegetative development</th>
<th>% Moisture content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fresh foliage, annuals developing early in growing cycle</td>
<td>300</td>
</tr>
<tr>
<td>Maturing foliage, still developing with full turgor</td>
<td>200</td>
</tr>
<tr>
<td>Mature foliage, new growth complete and comparable to older perennial foliage</td>
<td>100</td>
</tr>
<tr>
<td>Entering dormancy, coloration starting, some leaves may have dropped from stem</td>
<td>50</td>
</tr>
<tr>
<td>Completely cured</td>
<td>Less than 30, treat as a dead fuel</td>
</tr>
</tbody>
</table>

*Figure 6.3: Live Fuel Moisture*
In problem 54, use Figure 6.3 and the formula \( m = 100 \left( \frac{w - d}{d} \right) \), where \( m \) = fuel moisture content percentage, \( w \) = net wet weight, and \( d \) = net dry weight to answer the questions.

54. A fuel sample collected in the field weighs 377 grams. After the sample is dried in an oven, it weighs 198 grams.
   a) Use the moisture content formula given above to find the live fuel moisture content.
   b) Use your answer in (a) and the table above to determine the stage of vegetative development for the fuel sample.

3. APPLICATIONS FOR ACCOUNTING

In problems 55 – 56, write an equation and solve to answer the questions.

55. The rental fee for a van is $40 per day plus $0.08 per mile. If Jan needs a van for two days, and she has $150 to spend on the rental, how many miles can she drive and stay within her budget?

56. Prescott manufactures its products at a cost of $4 per unit and sells them for $10 per unit. If the firm’s fixed cost is $12000 per month, how many units must Prescott sell to break even?

In problem 57, use the given equations to answer the question.

57. The management of Robertson Controls must decide between two manufacturing processes for its model C electronic thermostat. The monthly cost of the first process is given by \( C_1 = 20x + 10000 \) dollars, where \( x \) is the number of thermostats produced; the monthly cost of the second process is given by \( C_2 = 10x + 17400 \) dollars.
   a) If the projected sales are 800 thermostats at a unit price of $40, which process should management choose in order to maximize the company’s profit? Support your answer.
   b) What would the projected number of sales have to be in order for the costs for each process to be the same?

In problems 58 – 70, use the equations given in section III to answer the questions.

58. Suppose that you invest $1700 in an account that earns simple interest at an APR of 7.4%. Determine the accumulated balance after 8 years.

59. Suppose that you invest $21,000 in an account that earns interest at an APR of 4.1%, compounded annually. Determine the accumulated balance after 6 years.

60. Suppose that you invest $324 in an account that earns interest at an APR of 5.9%, compounded quarterly. Determine the accumulated balance after 22 years.

61. Suppose that you invest $6348 in an account that earns interest at an APR of 3.7%, compounded monthly. Determine the accumulated balance after 10 years.
62. Suppose that you invest $1637 in an account that earns interest at an APR of 6.2%, compounded continuously. Determine the accumulated balance after 7 years.

63. Suppose that you want to have a $90,000 retirement fund after 25 years. How much will you need to deposit now if you can obtain an APR of 10.2%, compounded daily? Assume that no additional deposits are to be made to the account.

64. Suppose that you want to have $45,000 in a college fund for your son after 18 years. How much will you need to invest now if you can obtain an APR of 6.89%, compounded continuously? Assume no additional deposits are to be made.

65. Your savings account pays an APR of 5.2%, compounded annually. If you deposit $700 at the end of each year for 8 years, what will be the accumulated balance in the account?

66. Suppose you set up a new IRA (individual retirement account) that pays an APR of 7%, compounded monthly. If you contribute $160 per month for 17 years, how much will the IRA contain at the end of that time?

67. Suppose you want your daughter’s college fund to contain $145,000 after 15 years. If you can get an APR of 9.3%, compounded monthly, how much should you deposit at the end of each month?

68. Suppose you have 18 months in which to save $3100 for a vacation cruise. If you can earn an APR of 4.3% compounded monthly, how much should you deposit at the end of each month?

69. Suppose that you have a balance of $5000 on a credit card in which you are charged 21% APR. If you want to pay off the balance in 3 years, calculate your monthly payment and total payment over the entire term. Assume you make no additional charges to the card.

70. Suppose you need to take out a $310,000 loan to buy a home. If you can get a 30-year loan at a fixed rate of 5.375%, how much will your monthly loan payments be? Note: this amount does not account for taxes and insurance.

4. APPLICATIONS FOR CULINARY ARTS

FUN FACT: In the restaurant business, the monthly cost of food sold can be calculated by using the following formula:

\[
\text{Monthly Cost of Food Sold} = \text{Monthly Food Purchases} + [(\text{Beginning Inventory}) - (\text{Ending Inventory})]
\]

In problems 71 – 73, use the formula above to calculate the “Cost of Food Sold”. Then calculate the “Food Cost %” for the month given. Recall: The food cost percentage of the total dollar amount of food sales is the cost of food.

71. Inventory Value March 31- $12,500
   Purchases April 1 – 30 $28,000
   Inventory Value April 30 $14,000
   Food Sales April 1 - 30 $90,500
   a) Cost of Food Sold (for April) = $
   b) Food Cost % (for April) = _________%
72. Using the information below, calculate the cost of food sold, and the cost of food sold % (Food Cost %) for the months of April and May. Round each to the nearest hundredth of a percent.

<table>
<thead>
<tr>
<th>Date</th>
<th>Food Inventory</th>
<th>Food Sales</th>
<th>Food Purchases</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 1</td>
<td>$12,450</td>
<td>April $285,600</td>
<td>April $92,325</td>
</tr>
<tr>
<td>May 1</td>
<td>$11,620</td>
<td>May $251,800</td>
<td>May $85,250</td>
</tr>
<tr>
<td>June 1</td>
<td>$12,740</td>
<td>June $253,200</td>
<td>June $82,760</td>
</tr>
</tbody>
</table>

$ COST OF FOOD SOLD

FOOD COST %

a) APRIL $___________________   ______.___%

b) MAY $___________________   ______.___%

73. Calculate the value of “beverages issued” (cost of beverages sold) to the bar given the following information:

Opening inventory  $9,000
Purchases   $12,000
Closing Inventory  $8,000
“Beverages Issued” = _______

FUN FACT: As a restaurant owner it is necessary to determine the cost of ingredients that have to be trimmed and cleaned. In doing so it is necessary to factor in the cost of the trim so that one can account for the total cost. By excluding the cost of the peels, pits, cores, seeds, rinds, and/or outer leaves one may underestimate the actual cost to prepare a menu item. As a result, a restaurant owner may run the risk of undercharging for the menu item and not covering the restaurants expenses. To ensure that all of the expenses are covered, the as-purchased cost and the edible portion cost must be considered. The as-purchased cost (APC) is the cost paid to the supplier for the non-fabricated (un-cleaned) ingredient. The edible portion cost (EPC) is the cost per unit of the fabricated (cleaned) ingredient. The EPC accounts not only for the cost of the fabricated product but also for the cost of the trim. It is important to note that the EPC is always greater than the APC unless there is no trim. In this case the APC is the same as the EPC. This means that the yield percent (the ratio of edible portion weight to as-purchased weight) is 100%. The lower the yield percent, the higher the EPC.

To calculate the edible portion cost (EPC) we use the following formula:

\[ EPC = \frac{APC}{\text{Yield percent (in decimal form)}} \]
In problems 74 – 75, use the edible portion cost formula above and the given information to answer the questions.

74. Portion size: 5 oz.
   Yield %: 90%
   A.P. Cost: $3.89 per pound
   a) Calculate the EPC of chicken breasts.
   b) Calculate the EPC per portion. (1 lb. = 16 oz.)

75. The following ingredients and quantities are for a Chicken Marsala recipe that yields 20 portions. Calculate the total cost and the cost per portion.

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Amount</th>
<th>Unit</th>
<th>A.P. cost</th>
<th>Unit</th>
<th>Yield %</th>
<th>Ingredient cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Butter</td>
<td>5 oz.</td>
<td>lb</td>
<td>$3.50</td>
<td>100%</td>
<td>$_______</td>
<td></td>
</tr>
<tr>
<td>b) Chicken</td>
<td>100 oz.</td>
<td>lb</td>
<td>$68.00</td>
<td>70%</td>
<td>$_______</td>
<td></td>
</tr>
<tr>
<td>c) Mushrooms</td>
<td>2 1/4 lb</td>
<td>lb</td>
<td>$33.50</td>
<td>85%</td>
<td>$_______</td>
<td></td>
</tr>
<tr>
<td>d) Flour</td>
<td>4 oz.</td>
<td>lb</td>
<td>$12.75</td>
<td>100%</td>
<td>$_______</td>
<td></td>
</tr>
<tr>
<td>e) White stock</td>
<td>3 1/2 cups</td>
<td>lb</td>
<td>$58.00</td>
<td>100%</td>
<td>$_______</td>
<td></td>
</tr>
<tr>
<td>f) Marsala</td>
<td>400 ml</td>
<td>6/ltr</td>
<td>$75.00</td>
<td>100%</td>
<td>$_______</td>
<td></td>
</tr>
<tr>
<td>g) TOTAL COST</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$_______</td>
<td></td>
</tr>
<tr>
<td>h) Cost per portion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$_______</td>
<td></td>
</tr>
</tbody>
</table>

In problem 76, use the EPC formula to help input calculations for all shaded cells.

76. Menu Item: Carrot and Ginger Soup

<table>
<thead>
<tr>
<th>QTY</th>
<th>Unit</th>
<th>Ingredient</th>
<th>Invoice</th>
<th>Yield</th>
<th>EP Cost per Ingredient</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>lb.</td>
<td>Carrots</td>
<td>$20.75</td>
<td>85%</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>oz.</td>
<td>Clarified butter</td>
<td>$58.00</td>
<td>80%</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>lb.</td>
<td>Onions</td>
<td>$18.00</td>
<td>90%</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>lb.</td>
<td>Ginger</td>
<td>$12.00</td>
<td>82%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>oz.</td>
<td>Garlic</td>
<td>$3.00</td>
<td>75%</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>gal</td>
<td>Chicken stock</td>
<td>$40.00</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>each</td>
<td>Lemons</td>
<td>$13.00</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>qt.</td>
<td>Heavy Cream</td>
<td>$38.00</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>oz.</td>
<td>Dill</td>
<td>$16.00</td>
<td>40%</td>
<td></td>
</tr>
</tbody>
</table>

Total Recipe Food Cost

Food Cost per Portion
In problem 77, answer the questions.

77. A restaurant has fixed costs of $200,000 and Variable costs of 70% of the check average, (with no semi-variable costs). The check average is $12.50.
   a) Write an equation that describes the revenue the restaurant receives from each customer. Let x = # of meals served, R = revenue.
   b) Write an equation that describes the restaurants costs. Let x = # of meals served, C = Cost.
   c) Write an equation for the restaurants net. Let x = # of customers, N = net.
   d) Use your equation in (c) to determine how many meals are required to break even?
   e) How much total sales must the restaurant achieve to break even?

5. APPLICATIONS FOR EARLY CHILDHOOD EDUCATION

In problems 78 – 88, solve the common geometry formulas for the indicated variable. In some cases, you may need to use the quadratic formula:

78. Solve for h: \( A = \frac{bh}{2} \)
79. Solve for s: \( P = 2b + 2s \)
80. Solve for a: \( A = \frac{1}{2} h(a + b) \)
81. Solve for r: \( C = 2\pi r \)
82. Solve for n: \( D = \frac{n(n - 3)}{2} \)
83. Solve for n: \( S = 180(n - 2) \)
84. Solve for w: \( V = lwh \)
85. Solve for L: \( A = 2Lh + 2wh \)
86. Solve for s: \( L = 4s^2 \)
87. Solve for \( \pi \): \( V = \frac{1}{3} \pi r^2 h \)
88. Solve for r: \( S = 2\pi r^2 + 2\pi rh \)

6. APPLICATIONS FOR GRAPHIC DESIGN/PROFESSIONAL PHOTOGRAPHY

In problems 89 – 90, write an equation and solve to answer each question.

89. Byron is a wedding photographer. He develops two different-size prints for a newlywed couple. He sells the larger print for $7 each and the smaller print for $3 each. He develops twice as many smaller prints as larger prints at a total cost of $78. How many of each print did he sell?

90. Ilse decides to publish a children’s book. She calculates that it costs $1125 to print and market her book on the internet. If Ilse’s marketing agency sells the book for $14.99, and
Ilse’s revenue is equivalent to 25% of the cost of the book for each book sold, how many books must Ilse sell to break even? How many books must she sell so that her revenue doubles her costs?

FUN FACT: There are approximately 300 ink manufacturing companies (for printing) in the U.S. and many more throughout the industrial nations of the world. The following formulas address ink needs and costs for the industry of graphic communications.

**Ink Coverage Formula:**

\[
\text{Overall Image Area (in square inches) } \times \text{ Percent Coverage (in decimal form) } \times \text{ Number of Copies } = \\
\text{Ink Coverage Needs (in square inches)} \\
A \cdot r \cdot n = I
\]

FUN FACT: Different types of colors of ink will cover different amounts of area according to the type of paper being printed upon. The *ink mileage factor* is a number that determines how much area a certain color of ink will print from one paper to another. Mileage factor charts can be used to determine the ink mileage factor.

**Ink Need Formula:**

\[
\frac{\text{Overall image area}(1 + \text{ Percent Ink Waste (as decimal)})}{\text{Mileage Factor}} = \text{Pounds of Ink Needed} \\
\frac{A(1 + r)}{m} = w
\]

**Ink Cost Formula:**

\[
\text{Number of Pounds of Ink Needed } \times \text{ Selling Price } + \text{ Shipping Cost } = \text{Total Cost of Ink} \\
w \cdot p + c = T
\]

In problems 91 – 93, use the appropriate formula given above to answer the question. Round each answer to the nearest hundredth, if necessary.

91. 500 standard 8.5” by 11” pages will be used to print a job in which each page has 15% coverage. How many square inches of ink coverage is required for the job?

92. A printing company is printing 425,000 square inches of image area using regular black ink on litho coated paper that has a mileage factor of 380,000 square inches. If there is 3% of wasted ink, how many pounds of ink is needed to complete the job?

93. If 1.15197 pounds of ink is needed at $15.35 per pound of ink with a shipping cost of $5.50, what will be the total cost of the ink?
FUN FACT: For 35mm cameras, a **camera bellows** is an adjustable width extension tube. Any extension of the lens causes a reduction in the brightness of the image that must be adjusted for. Since the light must travel further, there needs to be more of it, therefore corrections in exposure settings ("bellows adjustments") are necessary to compensate when the distance from lens to film plane is increased beyond the focal length of the lens. The adjustments made to modern cameras are known as **f-stop adjustments**. In the following problems we will be using formulas to help determine the **bellows factor** and f-stop adjustments.

**Bellows Factor:**

\[
\text{Bellows Factor} = \frac{(\text{Bellows Extension})^2}{(\text{Length of Lens})^2}
\]

\[
B = \frac{E^2}{L^2}
\]

Note: In the bellows factor formula, the length of the bellows extension and the length of the lens must have the same units. It is common to need to convert the length of the lens from millimeters to inches, since the length of the bellows extension is in inches.

FUN FACT: “LOG” on your calculator refers to the **common logarithm** which is a base 10 logarithm (\(y = \log_{10} x\)). “LN” on your calculator refers to the **natural logarithm** in which the base is the natural base \(e \approx 2.718\). When we see the expression \(\log_b x\) we should be thinking: To what power do we raise \(b\) to get \(x\)? We will use the common logarithm in this next formula for calculating f-stop adjustments.

**F-Stop Adjustment:**

\[
F - \text{Stop Adjustment} = \frac{\log (\text{Bellows Factor})}{0.3}
\]

\[
F = \frac{\log B}{0.3}
\]

In problems 94 – 96, use the Bellows factor formula and F-Stop Adjustment formula to answer the questions. Round each bellows extension answer to the nearest thousandth, and each f-stop adjustment answer to the nearest quarter of an f-stop.

94. A camera with a 210 mm lens has a bellows extension of 15.25 inches.
   a) Calculate the bellows extension factor.
   b) Calculate the f-stop adjustment.

95. A camera with a 90 mm lens has a bellows extension of 6 inches.
   a) Calculate the bellows extension factor.
   b) Calculate the f-stop adjustment.
96. A camera with a 300 mm lens has a bellows extension of 14 inches.
   a) Calculate the bellows extension factor.
   b) Calculate the f-stop adjustment.

7. APPLICATIONS FOR INTEGRATED ENERGY TECHNOLOGY

FUN FACT: To determine the total cost for a solar photovoltaic (PV) system to be installed we must know the **array size** (that is, total number of solar modules or panels needed) in kilowatts for the location in which we plan to install them. To calculate the array size, we need to know, on average, how many hours per day the sun shines on our location. In Figure 6.4, we are given a solar map of the United States. We can use this map to find the average solar hours per day at our home. Also, due to real world efficiency losses (irradiance, dust, temperature, and wiring), we should expect our system power output (AC power) to be about 75% of the system (DC power) size.

![Solar Map](image)

**Figure 6.4: U.S. solar map for number of hours the sun shines per day in kW at each location**

**Array Size Formula:**

\[
Array\ Size\ (in\ kW) = \frac{4 \cdot (#\ of\ kWh\ per\ year\ used\ on\ average\ at\ home) \cdot (percent\ of\ energy\ bill\ to\ cover)}{1095 \cdot (solar\ hours\ per\ day\ (in\ kW)\ at\ home)}
\]

\[
A = \frac{4k r}{1095s}
\]

Note: The percent of energy bill to cover, \(r\), should be in decimal form.
In problems 97 – 99, use the array size formula to answer the following questions. Round each answer to the nearest hundredth.

97. Jenny wants to install a solar PV system to cover 80% of her energy bill. If Jenny uses approximately 8760 kWh of energy per year and her home receives approximately 3.7 kW of solar hours per day, what size PV system should she use?

98. Brooklyn wants to install a solar PV system to cover 95% of her energy bill. If Brooklyn uses approximately 10,600 kWh of energy per year and her home receives approximately 6.3 kW of solar hours per day, what size PV system should she use?

99. Justin wants to install a solar PV system to cover 65% of his energy bill. If Justin uses approximately 9650 kWh of energy per year and his home receives approximately 2.4 kW of solar hours per day, what size PV system should he use?

**FUN FACT:** The German scientist Georg Simon Ohm (1789 – 1854) studied electricity and in doing so discovered the relationship between power, voltage, current, and resistance. Ohm’s Law for voltage is the equation that is used to determine voltage, \( V \) in volts, given a current, \( i \) in amperes (or amps), and a resistance, \( r \) in ohms. **Voltage** is the electrical pressure in a circuit. **Current** is a measure of how the electricity is flowing. **Resistance** is the “resistance” to the flow of electricity through a wire. **Power**, \( P \) in watts, is how the voltage times the current. This means that a high number of volts flowing at a fast rate of current will produce a lot of watts.

**Ohm’s Law for Voltage:**
\[
V = i \cdot r
\]

**Ohm’s Law for Power:**
\[
P = i^2 \cdot r
\]

**Ohm’s Law for Electric Energy:**
\[
E = P \cdot t
\]

In problems 100 – 102, use Ohm’s Laws to answer the following questions.

100. A nine volt battery is flowing to a curling iron with a resistance of 18 ohms. How much current is flowing through the curling iron? How much power is needed to run the curling iron? How much electric energy is used to run the curling iron?

101. A 110 volt wall outlet supplies power to a strobe light with a resistance of 2200 ohms. How much current is flowing through the strobe light? How much power is needed to run the strobe light? How much electric energy is used to run the strobe light?
102. A CD player with a resistance of 40 ohms has a current of 0.1 amps flowing through it. How many volts supply the CD player? How much power is needed to run the CD player? How much electric energy is needed to run the CD player?

FUN FACT: The basic heat formula calculates the amount of heat needed, \( Q \) in BTU (British Thermal Unit) to heat a certain mass, \( m \) in pounds from one temperature, \( t \) in degrees Fahrenheit, to another temperature, \( T \) in degrees Fahrenheit. The specific heat, \( C \), is a constant that depends on the liquid that we are heating. If we are heating water, \( C = 1 \). If we are heating a water/glycol mix, \( C = 0.86 \).

\[
\text{Heat Formula:} \quad Q = mC(T - t)
\]

In problems 103 – 106, use the heat formula to answer the following questions. To heat water, use \( C = 1 \), and to heat a water/glycol mix use \( C = 0.86 \). Round each answer to the nearest tenth.

103. Calculate the heat needed to heat 41.5 pounds of water from 76°F to 93°F.
104. Calculate the heat needed to heat 82.3 pounds of a water/glycol mix from 68°F to 97°F.
105. It takes 2500 BTU’s to heat 67 pounds of water to a desired temperature. If the initial temperature is 64°F, what is the desired temperature?
106. It takes 4250 BTU’s to heat a certain number of pounds of water/glycol mix from 77.5°F to 88°F. How many pounds of water/glycol mix is being heated?

8. APPLICATIONS FOR PROCESS TECHNOLOGY

FUN FACT: Named after chemist and physicist Robert Boyle who published the original law in 1662, Boyle’s law is one of many gas laws and is a special case of the ideal gas law. Boyle’s law describes the inversely proportional relationship between the absolute pressure and volume of gas, if the temperature is kept constant within a closed system (i.e., a system that can exchange energy, such as heat or work, but not matter, with its surroundings).

\[
\text{Boyle’s Law:} \quad P_1 \cdot V_1 = P_2 \cdot V_2
\]

Where \( P_1 \) and \( V_1 \) are the pressure and volume of the 1st scenario, respectively, and \( P_2 \) and \( V_2 \) are the pressure and volume of the 2nd scenario, respectively, where the temperature is constant. Note: Make sure units match up!
In problems 107 – 111, use Boyle’s law to answer the following questions. Round each answer to the nearest thousandth, if necessary.

107. The volume of the lungs is measured by the volume of air inhaled or exhaled. If the volume of the lungs is 2.4 L during exhalation and the pressure is 101.7 kPa, and the pressure during inhalation is 101.01 kPa, what is the volume of the lungs during inhalation?

108. The total volume of a soda can is 415 mL. Of this 415 mL, there is 60 mL of headspace for the CO₂ gas put in to carbonate the beverage. If a volume of 100 mL of gas at standard pressure (101.325 kPa) is added to the can, what is the pressure in the can when it has been sealed?

109. It is hard to begin inflating a balloon. A pressure of 800 kPa is required to initially inflate the balloon 225 mL. What is the final pressure when the balloon has reached its capacity of 1.2 L?

110. If a piston compresses the air in the cylinder to 1/8 its total volume and the volume is 930 cm³ at standard pressure (101.325 kPa), what is the pressure after the gas is compressed?

111. If a scuba tank that has a capacity of 10 dm³ is filled with air to 500 kPa, what will be the volume of the air at 702.6 kPa?

9. APPLICATIONS FOR SKI AREA OPERATIONS

FUN FACT: In ski area planning, to logically plan or evaluate a ski area’s facilities, ski area management must be able to quantify the capacity of each facility. Once the capacities are identified in common terms, management can then evaluate internal capacities of various segments of their ski area as well as comparing the capacities with those of other ski areas. In this section we will introduce three formulas that are used to determine a comfortable carrying capacity at a ski area.

Passengers Per Hour (PPH): This is found by multiplying the number of people per carrier, \( p \), by 3600 seconds divided by the interval between carriers, \( i \), (in seconds). This is the theoretical PPH and does not include empty chairs, slows or stops by the ski lift. An efficiency factor can be included to get the actual average operating PPH by a percentage figure that reflects actual loading conditions. For this unit, however, we will ignore the efficiency factor.

\[
PPH = \frac{3600p}{i}
\]
**Vertical Transport Feet Per Hour (VTFH):** This is found by multiplying the PPH by the vertical rise of the lift. Vertical rise is the difference in vertical elevation from the base terminal, $h_1$, to the top terminal, $h_2$, measured in feet.

$$VTFH = PPH \cdot (h_2 - h_1) = \frac{3600p(h_2 - h_1)}{i}$$

**Skiers At One Time (SAOT):** This is found by multiplying the VTFH by the number of hours, $t$, of operation per day divided by the average daily vertical demand, $d$, for the type of skier served by the lift. Averages vary from ski area to ski area, but in general, demand for beginner, intermediate and advanced skiers are 2,000, 8,000, and 20,000 vertical feet of skiing, respectively.

$$SAOT = VTFH \cdot \frac{t}{d} = \frac{3600pt(h_2 - h_1)}{id}$$

In problems 112 – 114, use the PPH, VTFH, and SAOT formulas to answer the following questions. Round each answer to the nearest whole number.

112. A ski resort has a ski lift that begins at an elevation of 6540 feet and ends at an elevation of 10,542 feet that is primarily used by intermediate skiers with an average daily vertical demand of 9500 feet. Each of the chairs on the ski lift carry 4 passengers and the interval between chairs is 6.5 seconds. If the ski lift operates 7 hours per day,
   a) What is the PPH for this lift?
   b) What is the VTFH for this lift?
   c) What is the SAOT for this lift?

113. A ski resort has a ski lift that begins at an elevation of 5740 feet and ends at an elevation of 8650 feet that is primarily used by beginner skiers with an average daily vertical demand of 3400 feet. Each of the chairs on the ski lift carry 2 passengers and the interval between chairs is 9.4 seconds. If the ski lift operates 6 hours per day,
   a) What is the PPH for this lift?
   b) What is the VTFH for this lift?
   c) What is the SAOT for this lift?

114. A ski resort has a ski lift that begins at an elevation of 8925 feet and ends at an elevation of 11,470 feet that is primarily used by advanced skiers with an average daily vertical demand of 20,450 feet. Each of the chairs on the ski lift carry 4 passengers and the interval between chairs is 7 seconds. If the ski lift operates 8 hours per day,
   a) What is the PPH for this lift?
   b) What is the VTFH for this lift?
   c) What is the SAOT for this lift?
Solutions to Module VI:
1. $\frac{7}{6}$  2. $-1$  3. $-3$  4. 3  5. $-\frac{5}{4}$  6. $-2$  7. $-\frac{25}{8}$  8. $-\frac{36}{9}$  9. $\frac{1}{6}$  10. $-\frac{5}{239}$  11. 4  12. 1  13. 0  14. $\frac{5}{6}$  15. $-\frac{6}{5}$  16. $r = \frac{A-P}{Pt}$  17. $P = \frac{A}{1 + rt}$  18. $r = \frac{S-a}{S}$  19. $x = \frac{a+5}{2}$  20. $x = \frac{-By-C}{A}$  21. $a = \frac{b-c}{5}$  22. $p = \frac{mn}{40t}$  23. $n = \frac{7mk-2k}{3}$  24. $v = \frac{x-s}{t}$  25. $x + (x + 4) = 34$; 15, 19  26. $x + x + 4x + 4x = 300$; 120 ft. by 30 ft.  27. $x + (x + 2) = 72$; 35 in., 37 in.  28. $x + (x + 1) + (x + 2) = 66$; 21, 22, 23  29. $x + (x + 2) + (x + 4) = 117$; 37, 39, 41  30. $0.55x + 34.50 = 150$; 210 miles  31. $[\frac{1}{3}, \infty)$  32. $(-\infty, 2]$  33. $[\frac{1}{3}, \infty)$  34. $(-\infty, \frac{7}{6})$  35. $(-2, \infty)$  36. $(-\infty, 0]$  37. $(-\infty, -3) \cup \left(\frac{1}{2}, \infty\right)$  38. $(-4, 4)$  39. Approximately 1.339 square meters  40. 2.008 square meters  41. 2.022 square meters  42. 39.3 mg  43. 315 mg  44. Engine needs to perform at 30 psi  45. Engine needs to perform at 150 psi  46. 18 chains  47. 12 chains  48. 4.8 chains/hr  49. 12 hours  50. 2912 pounds of upward force  51. 2560 pounds of upward force  52. No; the weight of the beams exceed the maximum weight that the cable can hold by 220 pounds of force.  53. Yes; the cables can support 825 pounds of force while the concrete slab weighs 800 pounds.
54. a) Approximately 90.4%  b) Closest to mature foliage, new growth complete and comparable to older perennial foliage.  55. 875 miles  56. 2000 units  57. a) $C_2$ is the more profitable process for 800 projected thermostats sold. b) 740 thermostats
58. $2706.40  59. $26,725.37  60. $1175.28  61. $9184.99  62. $2526.58
63. $7029.85  64. $13,019.69  65. $6732.38  66. $62,420.19  67. $372.93  68. $167.04
69. $188.38; $6781.68  70. $1735.91  71. a) $26,500  b) 29.3%  72. a) $93,155; 32.62%  b) $84,130; 33.41%
73. $13,000  74. a) $4.32  b) $1.35  75. a) $3.50  b) $97.14  c) $39.41  d) $12.75  e) $58.00  f) $75.00  g) $285.80  h) $14.29
76. Menu Item: Carrot and Ginger Soup
Total Yield 1 1/2 gallons  Portion Size: 4 OZ  Total Portions: _________64______

<table>
<thead>
<tr>
<th>Recipe</th>
<th>Ingredient</th>
<th>QTY</th>
<th>Unit</th>
<th>Invoice</th>
<th>Cost</th>
<th>Unit</th>
<th>Unit%</th>
<th>EP Cost per</th>
<th>Yield</th>
<th>Grade</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>lb.</td>
<td>Carrots</td>
<td>$20.75</td>
<td>25</td>
<td>lb.</td>
<td>85%</td>
<td>$0.98</td>
<td>$11.72</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>10</td>
<td>oz.</td>
<td>Clarified butter</td>
<td>$58.00</td>
<td>30</td>
<td>lb.</td>
<td>80%</td>
<td>$0.16</td>
<td>$1.51</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>4</td>
<td>lb.</td>
<td>Onions</td>
<td>$18.00</td>
<td>50</td>
<td>lb.</td>
<td>90%</td>
<td>$0.40</td>
<td>$1.60</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.25</td>
<td>lb.</td>
<td>Ginger</td>
<td>$12.00</td>
<td>10</td>
<td>lb.</td>
<td>82%</td>
<td>$1.46</td>
<td>$0.37</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>oz.</td>
<td>Garlic</td>
<td>$3.00</td>
<td>1</td>
<td>lb.</td>
<td>75%</td>
<td>$0.25</td>
<td>$0.75</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>gal.</td>
<td>Chicken stock</td>
<td>$40.00</td>
<td>10</td>
<td>gal.</td>
<td>100%</td>
<td>$4.00</td>
<td>$4.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>each</td>
<td>Lemons</td>
<td>$13.00</td>
<td>160</td>
<td>ct.</td>
<td>100%</td>
<td>$0.08</td>
<td>$0.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5</td>
<td>qt.</td>
<td>Heavy Cream</td>
<td>$38.00</td>
<td>4.5</td>
<td>gal.</td>
<td>100%</td>
<td>$2.11</td>
<td>$3.17</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>oz.</td>
<td>Dill</td>
<td>$16.00</td>
<td>3</td>
<td>lb.</td>
<td>40%</td>
<td>$0.83</td>
<td>$1.67</td>
<td></td>
</tr>
</tbody>
</table>

Total Recipe Food Cost  
Food Cost per Portion  
$25.10  
$0.39

77. a) $R = 12.50x$  b) $C = 8.75x + 200000$  c) $N = 3.75x - 200000$  d) 53,333 1/3 meals

78. $h = \frac{2A}{b}$  79. $s = \frac{P - 2b}{2}$  80. $a = \frac{2A - bh}{h}$  81. $r = \frac{C}{2\pi}$  82. $h = \frac{3\sqrt{8D + 9}}{2}$
83. $n = \frac{S}{180} + 2$  84. $w = \frac{V}{lh}$  85. $L = \frac{A - 2wh}{2h}$  86. $S = \frac{\sqrt{L}}{2}$  87. $\pi = \frac{3V}{r^2h}$
88. $x = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8\pi S}}{4\pi}$  89. 6 large and 12 small prints.  90. 300 books; 600 books
91. 7012.5 square inches  92. 1.15 pounds of ink needed  93. $23.18$
94. 3.417  b) 1.779 or 1 ¾ stops  95. a) 2.867  b) 1.525 or 1 ½ stops  96. a) 1.405  b) 0.492 or ½ stop  97. 6.92 kW  98. 5.84 kW  99. 9.55 kW  100. 0.5 amps; 4.5 watts  101. 0.05; 5.5 watts
102. 4 volts; 0.4 watts  103. 705.5 BTU’s  104. 2052.6 BTU’s  105. 101.3°F  106. 470.7 lb  107. 2.412 l  108. 168.875 kPa  109. 150 kPa  110. 810.6 kPa  111. 7.116 dm³
112. a) 2215 PPH  b) 8,865,969 VTFH  c) 6533 SAOT  113. a) 766 PPH  b) 2,228,936 VTFH  c) 3933 SAOT  114. a) 2057 PPH  b) 5,235,429 VTFH  c) 2048 SAOT
The word geometry dates back to the Ancient Greeks and literally means “earth measure”; “geo” meaning earth, and “metry” or “metri”, meaning measure. Geometry is used in medical imaging such as CAT scans (an X-ray image made using computerized axial tomography) and sonograms; firefighters use geometry to locate a position in the field as well as finding the slope of an incline; accountants use coordinate geometry to display financial data with graphs; professional chefs use geometry to find volume; early childhood educators teach children the language of geometry as well as symmetry and angles to help them describe designs and patterns; In the practice of graphic design, understanding basic geometry is critical to their success; photographers use geometry to help produce images that are visually pleasing; solar photovoltaic installer’s use geometry to help them position solar panels so that the sun's energy is optimized; oil industry technicians use geometry and angles in pipe fitting; and ski areas use geometry to find the slope of an incline and when designing jumps.

There are many branches of geometry. In this module, we will be focusing on Euclidean Geometry (geometry that adheres to Euclid’s axioms, more specifically, the parallel postulate) with respect to angles and triangles and in Module VIII, we will continue our study of Euclidean geometry with circles, polygons, and geometric solids.

I. The Language of Angles

Khan Academy Resources:
https://www.khanacademy.org/math/basic-geo/basic-geo-angle/angle-intro/v/angle-basics

https://www.khanacademy.org/math/geometry-home/geometry-angles/angle-types/v/recognizing-angles-examples


The following are terms that are frequently used in geometry and angles.

1. **Line**: A line is determined by two distinct points and extends indefinitely in both directions. Lines in the Euclidean plane are either parallel, or they intersect in one point. Below is the line \( \overline{AB} \). The notation that is used to describe the line \( \overline{AB} \) is \( \overline{AB} \).

   ![Line](image)

2. **Ray**: A ray starts at a point and extends indefinitely in one direction. Below is the ray \( \overline{AB} \). The notation that is used to describe the ray \( \overline{AB} \) is \( \overline{AB} \).

   ![Ray](image)

3. **Line Segment**: A line segment is a part of a line and has two endpoints. Below is the line segment \( \overline{AB} \). The notation that is used to describe the line segment \( \overline{AB} \) is \( \overline{AB} \).

   ![Line Segment](image)

4. **Perpendicular Lines**: Perpendicular lines intersect to form right angles. The “box” in the corner indicates that the two lines form a right angle. The notation that is used to describe two perpendicular lines \( l_1 \) (line 1) and \( l_2 \) (line 2) is \( l_1 \perp l_2 \).

   ![Perpendicular Lines](image)

5. **Parallel Lines**: Lines that are parallel in Euclidean Geometry never intersect. The notation that is used to describe two parallel lines \( l_1 \) and \( l_2 \) is \( l_1 \parallel l_2 \).

   ![Parallel Lines](image)
6. **Plane**: A plane is a flat surface with no thickness and no boundaries. Think: a plane looks like a sheet of paper that extends infinitely in all directions.

![Diagram of a plane]

7. **Angle**: An angle (usually given in degrees “°”) is formed by two rays with the same endpoint called the vertex of the angle. The rays are called the sides of the angle. The notation that is used to describe the angle formed below is \( \angle CAB \) (angle C-A-B), or \( \angle BAC \) (angle B-A-C), or simply \( \angle A \) (angle A). Note: \( m\angle \) means “the measure of angle A”.

![Diagram of angle CAB]

8. **Right Angle**: A right angle measures 90°. The “box” in the corner indicates a right angle.

![Diagram of a right angle]

9. **Straight Angle**: A straight angle measures 180°.

![Diagram of a straight angle]

10. **Complementary Angles**: Complementary angles are two positive angles whose sum is 90°. In the figure below, \( \angle CAB \) and \( \angle DAC \) are complementary.

![Diagram of complementary angles]
11. **Supplementary Angles:** Supplementary angles are two positive angles whose sum is 180°. In the figure below, \( \angle CAB \) and \( \angle DAC \) are supplementary.

12. **Acute Angle:** An acute angle is an angle whose measure is between 0° and 90° (exclusive). In the figure below, \( \angle CAB \) is an example of an acute angle.

13. **Obtuse Angle:** An obtuse angle is an angle whose measure is between 90° and 180° (exclusive). In the figure below, \( \angle CAB \) is an example of an obtuse angle.

14. **Adjacent Angles:** Two angles that have a common vertex and a common side but have no interior points in common are adjacent angles. In the figure below, \( \angle CAB \) and \( \angle DAC \) are adjacent angles.
15. **Vertical Angles**: Two angles that are on opposite sides of the intersection of two lines are vertical angles. Vertical angles are equal in measure. In the figure below, $\angle A$ and $\angle B$ are vertical angles. Note: $\angle C$ and $\angle D$ are also vertical angles.

![Vertical Angles Diagram]

16. **Alternate Interior Angles**: Alternate interior angles are equal in measure. In the figure below, $l_1 \parallel l_2$ and $t$ is a transversal. $\angle A$ and $\angle B$ are alternate interior angles. Note: $\angle C$ and $\angle D$ are also alternate interior angles.

![Alternate Interior Angles Diagram]

17. **Alternate Exterior Angles**: Alternate exterior angles are equal in measure. In the figure on the previous page, $l_1 \parallel l_2$ and $t$ is a transversal. $\angle E$ and $\angle F$ are alternate exterior angles. Note: $\angle G$ and $\angle H$ are also alternate exterior angles.

18. **Corresponding Angles**: Corresponding angles are equal in measure. In the figure below, $l_1 \parallel l_2$ and $t$ is a transversal. $\angle A$ and $\angle H$ are corresponding angles. Note, there are three more sets of corresponding angles: $\angle E$ and $\angle D$, $\angle C$ and $\angle F$, and $\angle G$ and $\angle B$.

![Corresponding Angles Diagram]
Example 1: For the following, use your knowledge of angles to find the measure of angle $x$:

a) Given, $\angle LON = 90^\circ$

b) 

\[
\begin{align*}
7x - 3^\circ & \quad 4x + 18^\circ \\
\end{align*}
\]

c) If $l_1 \parallel l_2$, find $m\angle a$ and $m\angle b$:

\[
\begin{align*}
l_1 & \quad 47^\circ \\
\end{align*}
\]

Solution to Example 1:

a) Since $\angle LON = 90^\circ$ we can say that $\angle LOM$ and $\angle MON$ are complementary angles, that is, $\angle LOM + \angle MON = 90^\circ$. So we get the equation $(x) + (x + 18) = 90$. Solving for $x$ we get $x = 36^\circ$. Answer: $36^\circ$

b) Two intersecting lines form vertical angles, which are equal in measure. So we get the equation $(7x - 3) = (4x + 18)$. Solving for $x$ we get $x = 7^\circ$. Answer: $7^\circ$

c) $l_1 \parallel l_2$ is cut with a transversal, therefore we have alternate interior angles. $m\angle b = 47^\circ$ by alternate interior angles. Now, $\angle b$ and $\angle a$ are supplementary angles, that is, their sum is $180^\circ$. $m\angle a = 180^\circ - 47^\circ = 133^\circ$. Answer: $m\angle a = 133^\circ$ and $m\angle b = 47^\circ$
II. Properties of Triangles

Khan Academy Resources:

https://www.khanacademy.org/math/geometry-home/congruence/working-with-triangles/v/equilateral-and-isosceles-example-problems

In this section we will begin by defining a triangle, then, we will investigate different types of triangles that are defined by the measures of their interior angles as well as sides.

Three lines intersecting in three distinct points (vertices) forms a **triangle**. The following three properties hold true for all triangles.

1. The angles within the region enclosed by the triangle are **interior angles**. Their sum is 180°. In the figure below, \( m\angle A + m\angle B + m\angle C = 180° \).

   \[
   \begin{array}{c}
   C \\
   A \\
   B \\
   \end{array}
   \]

2. The sum of the lengths of any two sides of a triangle must exceed the length of the remaining side. This is known as the **triangle inequality**.

3. Larger angles are opposite larger sides.

**Example 2:** Given that \( m\angle y = 110° \), find \( m\angle a \) and \( m\angle b \):

\[
\begin{array}{c}
\text{y} \\
\text{a} \\
\text{b} \\
\end{array}
\]
Solution to Example 2:
Since angle y is 110° we can find one of the interior angles of the triangle that is formed by
supplementary angles. Now we have two interior angles of the triangle, 70° and 90° (since
there is a right angle). Now we can find b. Since the sum of all of the interior angles of a
triangle is 180°, we can subtract the sum of 70° and 90° from 180° to get b. b = 20°. Now to
find a, we know that angles a and b are supplementary, so a = 160°. Answer: a = 160°, b = 20°

Types of Triangles:

• **Isosceles Triangle:** Exactly two sides have equal length and the angles opposite the
equal sides are of equal measure.

![Isosceles Triangle Diagram]

• **Equilateral Triangle:** All sides are of equal length and all angles of equal measure (60°).

![Equilateral Triangle Diagram]

• **Scalene Triangle:** A scalene triangle is a triangle in which no two sides are of equal
length and no two angles are of equal measure. In other words, no angles or sides are
the same in the scalene triangle.

• **Acute Triangle:** A triangle in which all three interior angles are acute angles.

• **Obtuse Triangle:** A triangle that contains one obtuse interior angle.

• **Right Triangle:** A triangle that contains a right interior angle.

• **Isosceles Right Triangle:** An isosceles triangle that contains a right interior angle.

### III. Similar Triangles


**Similar Triangles:** If \( \triangle ABC \sim \triangle DEF \) then:

1. The ratio of lengths of corresponding sides are equal. We say that they are
   “proportional”. This means that if we wish to find the length of a missing side given two
   similar triangles, we can set up a proportion and solve.
2. The measures of corresponding angles are equal (or congruent).
3. The ratio of corresponding heights is equal to the ratio of corresponding sides.
Also, if two angles of one triangle are congruent to two angles of another triangle, then we can say that the two triangles are similar.

Note: The above holds true for similar polygons. We will discuss polygons in Module VII.

**Congruent Triangles:** (\(\cong\) means “congruent”) \(\triangle ABC \cong \triangle DEF\) if:

1. The three sides of one triangle are equal in measure to the corresponding three sides of a second triangle.
2. Two sides and the included angle of one triangle are equal in measure to two sides and the included angle of a second triangle.
3. Two angles and the included side of one triangle are equal in measure to two angles and the included side of a second triangle.

**Example 3:** Let \(\triangle ABC \sim \triangle DEF\) and the height of \(\triangle DEF\) is 1.1.

a) Find \(\overline{DE}\)

b) Find the height of \(\triangle ABC\).

Solution to Example 3:

a) Since triangle ABC is similar to triangle DEF, we know that the lengths of the corresponding sides are proportional. Setting up a proportion, to solve for \(\overline{DE}\) we get \(\frac{\overline{DE}}{2.7} = \frac{2.4}{7.2}\). Solving for \(\overline{DE}\) we get \(\overline{DE} = 0.9\). **Answer:** \(\overline{DE} = 0.9\)

b) Since triangle ABC is similar to triangle DEF, we know that the ratio of corresponding heights is equal to the ratio of corresponding sides. Setting up a proportion by using the height of triangle DEF given to be 1.1, we can solve for the height of triangle ABC, we’ll call it \(h\). The proportion becomes \(\frac{h}{1.1} = \frac{7.2}{2.4}\). Solving for \(h\) we get \(h = 3.3\) **Answer:** \(h = 3.3\)
Example 4: A tree casts a shadow 22 ft long while at the same time a 5 ft 10 in. man casts a shadow 7 ft 2 in. long. Estimate the height of the tree to the nearest inch.

Solution to Example 4:
Since the triangle formed from the tree and its shadow is similar to the triangle formed from the man and his shadow we can set up a proportion, but first let’s write all of the measurements in inches. 22 ft = 264 in., 5 ft 10 in. = 70 in., and 7 ft 2 in. = 86 in. Using these figures with \( h \) = height of the tree, our proportion is: \[
\frac{86}{70} = \frac{h}{264} = \frac{70}{86}.
\] Solving for \( h \) we get, \( h \) is approximately 215 inches or 17 ft 11 in. Answer: 17 ft 11 in.

Homework Set:

In problems 1 – 8, name each angle’s complement and supplement, if possible.

1. 20°
2. 63°
3. 110°
4. 157.2°
5. 220°
6. 181°
7. \( \frac{63}{7} \)°
8. \( \frac{89}{10} \)°

In problems 9 – 17, use the given information and the figures to answer the question.

9. Given that \( \ell_1 \parallel \ell_2 \),

\[\begin{array}{cccccc}
& a & b & c & d & e \\
\ell_2 & & & & & f \\
& g & h & & & \\
\ell_1 & & & & & t
\end{array}\]
a) Name any pair of alternate interior angles in the above figure.
b) Find the sum of the measures of angles $b$ and $h$.
c) Name any pair of vertical angles in the above figure.
d) Find the difference of the measures of angles $a$ and $e$.

10. Find the measure of angle $y$.

$$ \begin{align*}
109^\circ & \quad y \\
\end{align*} $$

11. Find the measure of angle $x$.

$$ \begin{align*}
2x & \quad 111^\circ \quad x - 15^\circ \\
\end{align*} $$

12. Use the diagram to the right to solve for the missing angle measure.

\begin{itemize}
  \item a) If $m\angle 1 = 48^\circ$ find the measure of $\angle 2$ \hspace{2cm} \begin{align*} 
\end{align*}
  \item b) If $m\angle 2 = 123^\circ$ find the measure of $\angle 4$ \hspace{2cm} \begin{align*} 
\end{align*}
\end{itemize}

13. Use the diagram below to solve for $x$ and $y$.

$$ \begin{align*}
(3x + 1)^\circ & \quad (2y - 10)^\circ \\
(5x - 5)^\circ & \quad (y + 10)^\circ \\
\end{align*} $$
14. Given that $m\angle b = 122^\circ$ and $m\angle x = 31^\circ$, find the measures of angles $a$ and $y$.

![Diagram](image1)

15. In the figure below, the measure of angle $a$ is $9^\circ$ less than half the measure of angle $b$. Find the measure of angle $a$ and angle $b$.

![Diagram](image2)

16. In the figure below, find the measure of angle $A$.

![Diagram](image3)

17. In the figure below, find the measure of angle $x$.

![Diagram](image4)
In problems 18 – 24, \( \ell_1 \parallel \ell_2 \) in the figure below. Use the figure to find the missing angles.

In the figure:
- \( \ell_1 \)
- \( \ell_2 \)
- Points: \( a, b, c, d, e, f, g, t \)
- Angle 128°

18. Find \( m\angle a \).
19. Find \( m\angle b \).
20. Find \( m\angle c \).
21. Find \( m\angle d \).
22. Find \( m\angle e \).
23. Find \( m\angle f \).
24. Find \( m\angle g \).

In problems 25 - 28, use properties of triangles to answer the questions.

25. An angle of an isosceles triangle is given to be 36°. Find the measures of the remaining two angles.
26. Find any angle of an equilateral triangle?
27. Is it possible for an isosceles right triangle to have an interior angle of 95°? Explain.
28. If in a scalene triangle you are only given one angle measure of 44°, is it possible to find either of the two remaining angles? Explain

In problems 29 – 32, use your knowledge of similar triangles and the diagram to answer the questions.

29. In the figure below \( \overline{BD} \parallel \overline{AE} \). If \( \overline{BD} = 6 \text{ in.}, \overline{AC} = 15 \text{ in.} \) and \( \overline{AE} = 8 \text{ in.} \) Find \( \overline{BC} \).
30. In the following figure: $\overline{MQ}$ and $\overline{NP}$ intersect at $O$. $\overline{NO} = 16 \text{ in.}, \overline{MN} = 12 \text{ in.}, \overline{PQ} = 4 \text{ in.} \quad \text{and} \quad \overline{MQ} = 26 \frac{2}{3} \text{ in.} \quad \text{Find the perimeter of } \triangle OQP$

![Diagram of a triangle with vertices M, O, N, P, and Q.]

31. A ten-foot lamppost casts a shadow sixteen feet long and at the same time a person casts a shadow six feet long. How tall is the person?

32. With 100 feet of string out, a kite is 64 feet above ground level. When the girl flying the kite pulls in 40 feet of string, the angle formed by the string and the ground does not change. What is the height of the kite above the ground after the 40 feet of string have been taken in?

Solutions to Module VII:

1. 70°; 160° 2. 27°; 117° 3. N/A; 70° 4. N/A; 22.8° 5. N/A; N/A; 6. N/A; N/A
7. 26 4/7°; 116 4/7° 8. 1/10°; 90 1/10° 9. a) (Answers Vary) d & e b) 180° c) (Answers Vary) b & c d) 0° 10. 109° 11. 28° 12. a) 132° b) 123° 13. x = 23°, y = 60° 14. a = 91°, y = 149° 15. a = 54°, b = 126° 16. A = 53° 17. x = 23° 18. 52° 19. 128° 20. 128° 21. 52° 22. 52° 23. 128° 24. 52° 25. 36° & 108° or 72° & 72° 26. 60° 27. No 28. No 29. 11.25 in. 30. 64 in. 31. 3.75 ft. 32. 38.4 ft.
Module VIII
Measurement Geometry: Part II

This module is a continuation of Module VII. In module VII we introduced angles, triangles, and their properties. In addition to triangles, this module will take a look at many other two- and three-dimensional geometric figures and define their properties. Then we will measure two-dimensional plane figures by finding the perimeter and area for each figure, and we’ll measure three-dimensional figures by finding volume, surface area, and lateral area for each figure. We will begin with two-dimensional geometric figures.

I. Types of Polygons

*Polygon:* A polygon (meaning “many-angled”) is a closed figure determined by three or more line segments that lie in a plane. The line segments that form the polygon are called its *sides.* A triangle is a three sided polygon and a *quadrilateral* is a four-sided polygon. The table below gives the names of polygons up to a ten-sided polygon.

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>Name of Polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
</tr>
<tr>
<td>7</td>
<td>Heptagon</td>
</tr>
<tr>
<td>8</td>
<td>Octagon</td>
</tr>
<tr>
<td>9</td>
<td>Nonagon</td>
</tr>
<tr>
<td>10</td>
<td>Decagon</td>
</tr>
</tbody>
</table>

*Regular Polygon:* A regular polygon is one in which each side has the same length and each angle has the same measure. All of the polygons discussed in this section are *convex,* that is, every interior angle is less than 180°. Otherwise, the polygon is *concave.* See Figure 8.1 below.

*Figure 8.1: Convex and Concave Polygons*
Types of Quadrilateral's:

1. **Rectangle:** In a rectangle, opposite sides are parallel, opposite sides are equal in length, all angles measure 90°, and the diagonals are equal in length. A diagonal is the line connecting one vertex to another passes through the “inside” of the polygon.

   ![Rectangle Diagram]

   \[ w = \text{width} \]
   \[ l = \text{length} \]

2. **Square:** A square is a rectangle in which all sides are of equal length.

   ![Square Diagram]

   \[ s = \text{side} \]

3. **Parallelogram:** In a parallelogram, opposite sides are parallel, opposite sides are equal in length, and opposite angles are equal in measure.

   ![Parallelogram Diagram]

   \[ h = \text{height} \]
   \[ s = \text{side} \]
   \[ b = \text{base} \]

4. **Rhombus:** A rhombus is a parallelogram in which all sides are of equal length.

   ![Rhombus Diagram]

5. **Trapezoid:** In a trapezoid, two sides are parallel.

   ![Trapezoid Diagram]

   \[ a = \text{parallel side} \]
   \[ h = \text{height} \]
   \[ b = \text{parallel side} \]
6. **Isosceles Trapezoid**: A trapezoid in which the nonparallel sides equal in length and the base angles are equal in measure.

![Isosceles Trapezoid]

**Circle**: A circle is the set of all points in a plane that are equidistant from a point called the center of the circle. The **diameter** of a circle is a line segment with endpoints on the circle that passes through the center of the circle. The **radius** of the circle is a line segment from the center of the circle to a point on the circle. Note: A circle is NOT a polygon.

![Circle]

**r = radius**

II. Measuring Polygons & Circles

Khan Academy Resources:

**Measuring Plane Figures**:
- **Perimeter**: Perimeter is the measure of the total distance around a plane geometric figure. Perimeter is measured in plain units, such as inches, feet, meters, etc.
- **Circumference**: Circumference is the perimeter (or distance around) a circle.
- **Area**: The amount of surface in a region or an enclosed plane geometric figure. Area is measured in “square units”, such as square inches (in²), square feet (ft²), square meters (m²), etc.

In most cases memorization of formulas are not necessary. For instance, if we remember that perimeter is the total distance around a plane geometric figure, we may simply add all of the sides in the given figure. For area, if we remember that the area of a rectangle is length × width (or base × height), we can easily find the area of a triangle since a triangle is a rectangle that is cut in half at its diagonal. Also, the area of a parallelogram is easy to calculate, since it is the same as the rectangle (base × height). To find the area of a trapezoid, we can think that a trapezoid is a figure comprised of two triangles, one with base a, and the other with base b, both with the same height, h. So the area of the trapezoid is ½ah + ½bh. If we factor out ½h, we get the formula for the area of a trapezoid: ½h(a + b). A circle, unfortunately, needs to be memorized.
The table below gives the formulas for perimeter and area:

<table>
<thead>
<tr>
<th>Plane Geometric Figure</th>
<th>Perimeter (P) or Circumference (C)</th>
<th>Area (A)</th>
</tr>
</thead>
</table>
| Triangle
  \(a, b, c = \text{sides}\)
  \(h = \text{height}\) | \(P = a + b + c\) | \(A = \frac{1}{2} bh\) |
| Rectangle
  \(l = \text{length}, w = \text{width}\) | \(P = 2l + 2w\) | \(A = lw\) |
| Square
  \(s = \text{side length}\) | \(P = 4s\) | \(A = s^2\) |
| Parallelogram
  \(b = \text{base}, h = \text{height}, s = \text{side}\) | \(P = 2b + 2s\) | \(A = bh\) |
| Trapezoid
  \(a, b = \text{parallel sides},\)
  \(h = \text{height},\)
  \(l = \text{slant height}\) | \((\text{Add the sides})\) | \(A = \frac{1}{2} h(a + b)\) |
| Circle
  \(r = \text{radius},\)
  \(d = \text{diameter},\) | \(C = 2\pi r\) or \(C = \pi d\) | \(A = \pi r^2\) |
| Any Regular Polygon
  \(n = \text{number of sides}\)
  \(l = \text{side length}\)
  \(s = \text{distance from center to vertex}\) | \(P = nl\) | \(A = \frac{1}{2} n \cdot \sin \left(\frac{360}{n}\right) \cdot s^2\) \(\text{“sin” = sine function}\) |

**Other Formulas Associated with Polygons:**

Number of diagonals, \(D\), in a convex polygon with \(n\) sides:
\[
D = \frac{n(n - 3)}{2}
\]

Sum of the interior angles, \(S\), of a polygon with \(n\) sides:
\[
S = 180^{\circ}(n - 2)
\]
Example 1: Find the circumference of a circle with a radius of 6 in. Round your answer to the nearest hundredth.

Solution to Example 1:
To find the circumference of a circle given its radius we will use the formula $C = 2\pi r$. Substituting $r = 6$ into the formula we get: $C = 2\pi(6) = 12\pi$. Multiplying we get the circumference is approximately 37.7 in. Answer: 37.7 in.

Example 2: Find the area of the shaded region in the figure below. ABCD is a square and the interior portion is a circle. Round your answer to the nearest hundredth.

Solution to Example 2:
To find the area of the shaded region we must subtract the area of the circle from the area of the square. The area of the square is $10 \times 10 = 100 \text{ m}^2$. The area of the circle requires the length of the radius which can be found by dividing 10 by 2, since 10 is the diameter of the circle. With a radius of 5, we get the area of the circle: $A = \pi(5)^2 = 25\pi$, or approximately 78.54 m$^2$. Subtracting the area of the circle from the area of the square we get: $100 - 78.54 = 21.46 \text{ m}^2$. Answer: 21.46 m$^2$
III. Types of Geometric Solids

A geometric solid is a three-dimensional figure in space. In this section we will define six different geometric solids: (1) Rectangular Solid, (2) Cube, (3) Sphere, (4) Right Circular Cylinder, (5) Right Circular Cone, and (6) Regular Pyramid.

Geometric Solids:

1. Rectangular Solid: A rectangular solid is a geometric solid in which all six sides, called faces, are rectangles.

   $h = \text{height}$  
   $l = \text{length}$  
   $w = \text{width}$

2. Cube: A cube is a rectangular solid in which the lengths of all edges are the same and each face is a square.

   $s = \text{length of edge}$

3. Sphere: A sphere is a geometric solid in which all of the points on the sphere are equidistant from a fixed point called the center. The diameter of a sphere is a line segment with endpoints on the sphere passing through the center of the sphere. The radius of a sphere is the line segment from the center to a point on the sphere.

   $r = \text{radius}$

4. Right Circular Cylinder: A right circular cylinder is a cylinder in which the bases are circles and they are perpendicular to the height.

   $r = \text{radius}$  
   $h = \text{height}$
5. **Right Circular Cone:** A right circular cone is a right circular cylinder in which one of its bases is shrunk to a single point, called the vertex.

![Diagram of a right circular cone]

- $l = \text{slant height}$
- $h = \text{height}$
- $r = \text{radius}$

6. **Regular Pyramid:** A regular pyramid is a pyramid in which the base is a regular polygon and the sides are isosceles triangles.

![Diagram of a regular pyramid]

- $l = \text{slant height}$
- $h = \text{height}$
- $s = \text{length of base}$

### IV. Measuring Geometric Solids

**Khan Academy Resources:** [https://www.khanacademy.org/math/basic-geo/basic-geo-volume-sa](https://www.khanacademy.org/math/basic-geo/basic-geo-volume-sa)

In this section we will define three different measurements of geometric solids: (1) Volume, (2) Surface Area, and (3) Lateral Area.

**Volume:** Volume is a measure of the amount of space occupied by a geometric solid. Volume is measured in cubic units, such as cubic inches ($\text{in}^3$), cubic feet ($\text{ft}^3$), cubic meters ($\text{m}^3$), etc.

For any right geometric solid with identical bases (for example, rectangular solid, cube, right circular cylinder), the volume can be found by multiplying the area of the base times the height. If the right geometric solid has only one base and comes to a point (for example, right circular cone and regular pyramid), then the volume is the area of the base times the height all divided by $3$.

**Surface Area:** The surface area of a geometric solid is the total area on the surface of the solid. **Lateral Area:** The lateral area of a geometric solid is the area around the sides (excluding the base(s)).
To find the surface area, simply add together the areas of all the faces that make up the geometric solid. To find the lateral area, simply add together the areas of all the faces excluding the base(s). Surface area and lateral area is measured in square units, just like any other “area” measurement.

The table below gives the formulas for volume, surface area, and lateral area:

<table>
<thead>
<tr>
<th>Geometric Solid</th>
<th>Volume (V)</th>
<th>Surface Area (S)</th>
<th>Lateral Area (L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular Solid</td>
<td>( V = lwh )</td>
<td>( S = 2lw + 2wh + 2lh )</td>
<td>( L = 2lh + 2wh )</td>
</tr>
<tr>
<td>Cube</td>
<td>( V = s^3 )</td>
<td>( S = 6s^2 )</td>
<td>( L = 4s^2 )</td>
</tr>
<tr>
<td>Sphere</td>
<td>( V = \frac{4}{3} \pi r^3 )</td>
<td>( S = 4\pi r^2 )</td>
<td>N/A</td>
</tr>
<tr>
<td>Right Circular Cylinder</td>
<td>( V = \pi r^2 h )</td>
<td>( S = 2\pi r^2 + 2\pi rh )</td>
<td>( L = 2\pi rh )</td>
</tr>
<tr>
<td>Right Circular Cone</td>
<td>( V = \frac{1}{3} \pi r^2 h )</td>
<td>( S = \pi r^2 + \pi rl )</td>
<td>( L = \pi rl )</td>
</tr>
<tr>
<td>Regular Pyramid</td>
<td>( V = \frac{1}{3} s^2 h )</td>
<td>( S = s^2 + 2sl )</td>
<td>( L = 2sl )</td>
</tr>
</tbody>
</table>

Example 3: Find the volume of a sphere with diameter 10 cm. Round your answer to the nearest hundredth.
Solution to Example 3:
The formula for the volume of a sphere is \( V = \frac{4}{3} \pi r^3 \). We need the radius of the sphere. We can find the radius by dividing the diameter by 2, giving us a radius of 5 cm. Substituting \( r = 5 \) into the formula we get: 
\[
V = \frac{4}{3} \pi (5)^3 = \frac{500\pi}{3} \approx 523.60.
\]
Answer: \( 523.60 \text{ cm}^3 \)

Example 4: Find the lateral area of the regular pyramid given below:

Solution to Example 4:
The formula that we need to use to find the lateral area is \( L = 2sl \). Substituting \( s = 12 \), and \( l = 10 \) into the formula we get: 
\[
L = 2(12)(10) = 60.
\]
Answer: \( 60 \text{ ft}^2 \)

Homework Set:

In problems 1 – 6, give the name of the polygon.

1. ![Polygon 1]

2. ![Polygon 2]

3. ![Polygon 3]
In problems 7 – 11, answer true or false.

7. A square is a rectangle.
8. A rectangle is a square.
9. A rhombus is a square.
10. A square is a rhombus.
11. A geometric figure that is both a rectangle and a rhombus is a square.

In problems 12 – 17, determine if the given figure is a polygon.

12.

13.

14.
In problems 18 – 25, find the perimeter or circumference. Round each answer to the nearest hundredth, if necessary.

18. 

19. ABCD is a parallelogram.

20. A square with side $m = \frac{2}{3}$.

21. A rectangle with length 8.7 ft and width 3 ft.

22. 

In problems 18 – 25, find the perimeter or circumference. Round each answer to the nearest hundredth, if necessary.
23. In the figure below, $AB \perp BC$ and $AH \parallel DE$.

24. A circle with radius 2.35 feet.
25. A circle with a diameter of 5 ¼ yd.

In problems 26 – 28, find the perimeter and area of each figure. Round each answer to the nearest hundredth, if necessary.

26. Given arc BC is a quarter-circle, and ABDE is a rectangle.

27. Given arc AB is a quarter-circle, arc CD a semi-circle, and ACDE a rectangle.
28. Given ABD a right triangle, arc AB a half-circle, arc BC a quarter-circle.

29. Find the area of a square whose perimeter is 40 cm.
30. Find the area of a circle whose circumference is $10\pi$ yd.
31. Find the perimeter of a rectangle whose area is 24 in$^2$ and width is 4 in.
32. Find the area of an equilateral triangle whose side is 6 ft. and height is $3\sqrt{3}$ ft.
33. Find the area of an isosceles triangle whose base is 6 m and height is 3 m.
34. Find the area of a square with a side $2\sqrt{3}$ ft.
35. Find the area of the shaded region in the figure below.

36. Jason plans to put up a rectangular fence around his home. If the property measures 39 feet 6 inches by 105 feet 3 inches, how many feet of fencing will Jason need?
37. A baseball diamond is the shape of a square with sides 90 feet in length. If a player hits three home runs and a double, what will be the total distance the player will run around the bases?
38. A painter charges $.75 per square foot for painting house exteriors. One end of a house is a rectangle surmounted by a triangle. The base of the rectangle is 32 feet, the height of the rectangle is 8 feet and the height of the triangle is 7 feet. There are three windows, each 2 feet by 3 feet in the side of the house. How much will the painter charge for painting this side of the house?
39. The distance around a circular plot of land is 223 feet. If a person were standing in the center of the plot, approximately how far would she have to walk to reach the edge?
40. Angie is training for the Olympics. She wants to train on her school track, which is a circular area with diameter 122.3 feet. Angie wants to run at least 5 miles a day. How many times does she need to run around her school track?

41. Rich patrols a game reserve for the Colorado DOW with side lengths of 14.6 km, 18 ¼ km, 15.23 km, and 22.01 km. How far does he travel on this patrol during a 5-day week if he patrols the reserve three times each day?

In problems 42 – 44, find the perimeter and area. Leave your answer in exact form and in terms of π.

42. For the figure shown to the right, ABCD is a rectangle
   And arc AD is a semi-circle:

   a. Find the area
   b. Find the perimeter

43. For the figure shown to the right, ABCD is a rectangle, and arc BC is a quarter-circle:

   a. Find the area.
   b. Find the perimeter.

In problems 44 – 55, answer the questions. Round each answer to the nearest hundredth, if necessary.

44. Why don’t we give a formula for lateral area of a sphere?
45. Find the lateral area of a regular pyramid whose base is a square with side 12 m, and whose slant height 8 m.
46. Find the lateral area of a regular pyramid whose base is a square with side 12 m, and whose slant height is 10 m.
47. Find the lateral area of a right circular cone with base radius 3 ft and slant height 7 ft.
48. Find the surface area of a right circular cone with diameter 8 m and slant height 5 m.
49. Find the surface area of a sphere with diameter 20 m.
50. Find the lateral area and surface area of a rectangular solid with length 16 ft, width 11 ft, and height 12.2 ft.
51. Find the lateral area and surface area of a cube with an edge 7.1 m.
52. Find the lateral area and surface area of a right circular cylinder with diameter 3.8 inches and height 7.6 inches.
53. Find the surface area of a right circular cylinder in which the circumference of the base is 6\(\pi\) ft. and the height is 8 ft.
54. Find the surface area of a right circular cone with a radius of 7 inches and a slant height of 25 inches.
55. Find the surface area of the cube shown to the right:
   (Note: the edge of the cube is \(4\sqrt{2}\) inches)

\[
\text{In problems 56 – 66, answer the questions. Round your answers to the nearest hundredth, if necessary.}
\]
56. Find the volume of a cube with edge 3.7 m.
57. Find the volume of a right circular cone with a radius 3 ft and height \(2\sqrt{10}\) ft.
58. Find the volume of a sphere with radius 1 \(\frac{1}{2}\) inches.
59. Find the volume of a right circular cylinder in which the diameter of the base is 16 \(\frac{1}{2}\) ft and the height is 6 \(\frac{3}{4}\) ft.
60. Find the volume of a rectangular solid with length 9 ft, width 8 ft, and height 7 ft.
61. Find the volume of a regular pyramid with base 24 ft, and height 12 ft.
62. What is the ratio of the volume of a right circular cone to the volume of a right circular cylinder with the same base and height?
63. Find the volume of a cube whose lateral area is 100 m\(^2\).
64. Find the volume of a right circular cone with diameter 10 ft and height 12 ft.
65. Find the volume of a cone with a radius of 6 in. and height 4 in.
66. Find the volume of a circular cylinder whose height is 5 m and lateral area is \(30\pi\) m\(^2\).

\[
\text{In problems 67 – 75, answer the questions. Leave your answer in exact form, that is, in terms of }\pi\text{ and radicals, if they exist.}
\]
67. Find the volume and surface area of the figure below.
68. For the sphere given below, the radius is 9 cm:
   a) Find the volume of the sphere. Leave your answer in terms of π and include units.
   b) Find the surface area of the sphere.

![Sphere diagram]

69. A cube has a volume of 64 cubic inches.
   a) Find the length of the side of the cube.
   b) Find the surface area of the cube.

70. Find the volume of the solid (A right circular cylinder with a hole of radius 2 m drilled through it).

![Cylinder diagram]

71. Find the volume of the figure given below (a rectangular solid with a cylindrical hole drilled through it). The length, width, and height of the rectangular solid is 8 in. by 4 in. by 5 in., and the radius of the cylinder is 1.5 in.

![Rectangular solid diagram]
72. Find the surface area of the solid. The height is 24 in.

73. Find the volume, surface area and lateral area of the following.

74. Find the volume, surface area and lateral area of the following. The slant height is 5 m.

75. Find the volume, surface area and lateral area of the following.

In problems 76 – 83, answer the questions. Round each answer to the nearest hundredth, if necessary.

76. At a non-alcoholic fraternity party Jesse is challenged to a speed milk-drinking contest. The host of the party and challenger only has two mugs, one that is in the shape of a right circular cylinder with a radius of 2.4 in. and a height of 10 in. and another Denver Nuggets
basketball mug shaped as a sphere with a radius of 3.4 in. Which mug should Jesse choose to drink from, if the loser has to streak across campus?

77. A rectangular office building has a length of 55 ft., width of 42 ft. and a volume of 404,250 cubic feet. What is the height of the building?

78. You wish to paint a wooden structure that has the shape of the figure below. One gallon of paint covers 216 square feet. If the radius and height of the cylinder is 5 ft. and 9 ft. respectively and the length, width and height of the rectangular solid is 18 ft. by 12 ft. by 8 ft., how many gallons of paint are needed for the job?

79. Suppose that the length, width and height of a rectangular solid each tripled in measurement. Does this mean that the volume will also increase by a factor of 3 (i.e., triple in volume)? If not, by what factor will the volume increase?

80. Jim wants to put up new Alex Rodriguez baseball wallpaper in his son’s room. The dimensions of the room are 20’ by 15’ by 8’. How much wallpaper should Jim purchase for this project?

81. Julia is decorating her daughter’s room with wallpaper. If the dimensions of the room are 23’ by 19’ by 8’, how much wallpaper will Julia need to buy?

82. For a party you are in charge of ordering the drinks. If the drinks come in a circular cylinder container whose lateral area is $20\pi$ square feet and height is 4 feet. How much liquid is in the container?

83. Find the volume of a regular pyramid with base length 3 feet and height 8 feet.

1. APPLICATIONS FOR EMT/MEDICAL ASSISTANT/NURSING
FUN FACT: Two-part gel capsules (as seen in the figure above) are in the shape of a right circular cylinder combined with a sphere (approximately a half-sphere on the top and half-sphere on the bottom). These types of capsules are supplied to the pharmaceutical manufacturer as closed units, then the two halves are separated and the capsule is filled with powder. After the medicine is compressed in the capsule, the other half of the capsule is pressed on leaving no medicine in the upper portion (half-sphere) of the capsule. If we wanted to approximate the volume of medicine, in mL, of one of these capsules, we could find the volume of the shape below:

$$V = \pi r^2 h + \frac{1}{2} \left( \frac{4\pi r^3}{3} \right)$$

Total Volume = Volume of the cylinder + ½ the volume of a sphere

The locked length of a two-part gel capsule is from top to bottom (including the half-spheres), so we would have to subtract twice the radius of the sphere from the locked length to get the height of the cylinder.

In problems 84 – 89, approximate the volume, in milliliters of the two-part gel capsule given the locked length and the external diameter in the table below. Use the conversion factor: 1 mm$^3$ = 0.001 mL. Round each answer to the nearest hundredth.

<table>
<thead>
<tr>
<th>Size</th>
<th>Volume (mL)</th>
<th>Locked Length (mm)</th>
<th>External Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>84.</td>
<td>11.1</td>
<td>4.91</td>
</tr>
<tr>
<td>4</td>
<td>85.</td>
<td>14.3</td>
<td>5.31</td>
</tr>
<tr>
<td>3</td>
<td>86.</td>
<td>15.9</td>
<td>5.82</td>
</tr>
<tr>
<td>2</td>
<td>87.</td>
<td>18</td>
<td>6.35</td>
</tr>
<tr>
<td>1</td>
<td>88.</td>
<td>19.4</td>
<td>6.91</td>
</tr>
<tr>
<td>00</td>
<td>89.</td>
<td>23.3</td>
<td>8.53</td>
</tr>
</tbody>
</table>

90. In problems 84 – 89 above, the answers that we’ve obtained for the volume of each two-part gel capsule are a little bit higher than the actual volume. Why do you suppose this is the case?
2. APPLICATIONS FOR FIRE SCIENCE

FUN FACT: In fire science, the burn perimeter is determined by adding the lengths of the various lines that enclose the black area, or burn area of a fire. Because fires often burn in unusual shapes, the area and perimeter of a fire can be approximated by assembling a combination of known shapes and lines. For instance, in the figure below, we can estimate the burn area by considering the quadrilateral and the triangle. Since the quadrilateral is closest to the shape of a rectangle, we can estimate the area by find the average of the opposite sides (42 & 65), and (71 & 58), giving us a rectangle with length 53.5 paces and width 64.5 paces.

In problems 91 – 93, use the burn area that is mapped out in the figure on the previous page to answer the following questions.

91. Bob paces a burn area having dimensions as shown above. His pace is 11 paces per chain. What is the perimeter of the burn in chains?
92. Which answer best approximates the area of the burn in square paces? (average of sides multiplied)
   i. 2232 sq paces
   ii. 3451 sq paces
   iii. 5683 sq paces
   iv. 500 sq paces
93. What is the best approximated area from (92) above in acres? Recall: 1 acre = 10 square chains
   i. 0.4 acres
   ii. 5 acres
   iii. 52 acres
   iv. 5166 acres
FUN FACT: The volume of a hose allows us to estimate how much water can be delivered to the fire and is important in firefighting. A hose is the shape of a right circular cylinder. The inner diameter of a hose is synonymous with the diameter of a circle.

In problems 94 – 95, answer the following questions. Round each answer to the nearest tenth.

94. Carla is filling the cylindrical tank 8 feet in diameter and 10 feet deep with a pump. She’s pumping at a rate of 13 gallons per minute (gpm). How long will it take to fill the tank?

95. A 100-foot length of 1-inch diameter hose is charged with water. How many gallons of water are in that length of hose?

In problems 96 – 99, find the volume of each of the given hose lengths, then use the table to answer problems 100 - 103. Use the conversion factor: 1 ft³ = 7.4805 gallons. Round each answer to the nearest tenth.

<table>
<thead>
<tr>
<th>Description ID = Inner Diameter</th>
<th>U.S. Measure Volume (Capacity)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>One hose length</strong></td>
<td></td>
</tr>
<tr>
<td>1-1/2-in ID x 100 ft</td>
<td>96.</td>
</tr>
<tr>
<td>1-in ID x 100 ft</td>
<td>97.</td>
</tr>
<tr>
<td>3/4-in ID x 50 ft</td>
<td>98.</td>
</tr>
<tr>
<td>5/8-in ID x 50 ft</td>
<td>99.</td>
</tr>
<tr>
<td><strong>Weight</strong></td>
<td></td>
</tr>
<tr>
<td>1 gal of water at 20°C</td>
<td>8.3 lb</td>
</tr>
</tbody>
</table>

Table 8.2 Water Handling Unit Estimations
In problems 100 – 103, use the table above to answer the following questions. Round each answer to the nearest tenth.

100. Sherman is out on a fire. His crew has a trunk line of 6 lengths of 100-foot, 1 & 1/2 inch hose. He needs to estimate the volume of water in this trunk line. What is a good estimate?

101. An engine company is pumping a progressive hose lay with 1-inch laterals every 100 feet. At 800 feet up from the engine, the trunk line breaks. The firefighters replace it, but they forget to shut off the gated wye valve above the broken hose. As a result, they accidentally drain ten 100-foot lengths of 1-1/2-inch hose and ten 100-foot lengths of 1-inch hose. How much water above the break was lost due to this mistake?

102. The tank on a Model 62 Engine is filled with 500 gallons of water at 20° C (68° F). How much weight does the water add to the weight of the engine?

103. Two 100-foot lengths of 1 & 1/2-inch cotton-synthetic hose weigh about 54 pounds total when dry. How much will the same hose weigh when fully charged with water at 20° C?

3. APPLICATIONS FOR CULINARY ARTS

In problems 104 – 106, answer the questions. Use the conversion factors: 1 ft³ = 1728 in³, 1 ft³ = 7.4805 gal, and 1 gallon = 128 fl-oz. Round each answer to the nearest hundredth.

104. An ice cream cone with base radius 1.25 in. and a height of 4 in. is filled with ice cream. What is the volume, in fluid ounces, of the ice cream in the cone?

105. If the same cone in problem (104) included one scoop of ice cream in the shape of a sphere with radius 1.5 in., what is the total volume, in fluid ounces, of ice cream?

106. If there is ½ gallons in a carton of ice cream, how many ice cream cones in problem (105) can be made with one carton of ice cream?

107. A right circular cylindrical container has a radius of 2.5 in., and a height of 5 in. How many ounces of liquid will the container hold?

In problems 108 – 109, answer the questions. Round each answer to the nearest hundredth.

108. Which of the following cartons has the greatest storage space?
   a) A cube shaped box with edge 2.9 feet
   b) A cylindrical container with radius 2 meters and height 2 feet?
   c) A rectangular carton with length 3.1, width 1.9, and height 4.3 feet?

109. Which of the cartons in problem (105) has the greatest area to be wrapped?

4. APPLICATIONS FOR GRAPHIC DESIGN/PROFESSIONAL PHOTOGRAPHY

FUN FACT: In professional photography, pixel dimensions can be used to calculate the maximum print size at a print output of a given number of pixels per inch. We can think of pixel dimensions as being the dimensions of a rectangle, and the total pixels as the area of the
rectangle. Therefore, to find the number of megapixels in an image we must use the conversion factor: 1 megapixel = 1 million pixels.

In problems 110 – 115, you are given a digital SLR cameras made by Canon and Nikon and their maximum pixel dimensions. Calculate the number of megapixels of each camera. Round each answer to the nearest megapixel dimension.

110. Canon 50D, APS-C: 4752 × 3168 pixels.
111. Canon EOS 5D Mark II, full frame: 5616 × 3744 pixels.
112. Canon EOS 7D, APS-C: 5184 × 3456 pixels.
113. Nikon D700, full frame: 4256 × 2832 pixels.
114. Nikon D300, APS-C: 4288 × 2848 pixels.

FUN FACT: To find the maximum print size, we must know the cameras pixel dimensions and then convert them to inches by using a given output resolution rate in pixels per inch (ppi). For instance, if a camera has pixel dimensions of 4500 × 3000, the maximum print size at 300 ppi would be 15 in. by 10 in. by dividing each of the pixel dimensions by 300.

In problems 116 – 119, calculate the maximum print size for the following cameras with an output resolution of 300 ppi. Round your answer to the nearest hundredth.

117. Nikon D3X, full frame: 6048 × 4032 pixels.
118. Canon 50D, APS-C: 4752 × 3168 pixels.
119. Canon EOS 5D Mark II, full frame: 5616 × 3744 pixels.

In problems 120 – 126, fill in the missing information. Round each megapixel dimension to the nearest megapixel and each print size dimensions to the nearest hundredth of an inch.

<table>
<thead>
<tr>
<th>Megapixels</th>
<th>Pixel Resolution</th>
<th>Print Size @ 300ppi</th>
<th>Print Size @ 200ppi</th>
<th>Print Size @ 150ppi</th>
</tr>
</thead>
<tbody>
<tr>
<td>120.</td>
<td>2048 x 1536</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>121.</td>
<td>2464 x 1632</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>122.</td>
<td>3008 x 2000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>123.</td>
<td>3264 x 2448</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>124.</td>
<td>3872 x 2592</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>125.</td>
<td>4290 x 2800</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>126.</td>
<td>4920 x 3264</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FUN FACT: The Greeks (during the time of Pythagoras 500 B.C.) were interested in the question: How can a line segment be divided into two pieces that have the most appeal and balance?

The Greeks claimed that the most visually pleasing division of the line had the property that the ratio of the length of the long piece to the length of the short piece is the same ratio of the length of the entire segment to the length of the long piece. The equation below defines this relationship:

\[
\frac{L}{l} = \frac{L + 1}{L}
\]

127. Solve for \( L \) in the equation: \( \frac{L}{l} = \frac{L + 1}{L} \).

To complete this you will need the quadratic formula for solving quadratic equations, but still give this problem a shot. **Quadratic Formula:** For \( ax^2 + bx + c = 0 \), \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

128. Which of the following rectangles seems to be the most pleasant to look at?

![Rectangle A](image1.png)

![Rectangle B](image2.png)

![Rectangle C](image3.png)

FUN FACT: The **golden rectangle** is a rectangle whose long side is \( \Phi = \frac{1 + \sqrt{5}}{2} \) times the height.

That is the ratio of the base to the height is \( \Phi \) to 1. The early Greeks considered rectangles like rectangle C to be the most pleasing to the eye. The golden rectangle is present in many famous structures (like the Parthenon) and art. And the golden ratio is present in nature.

FUN FACT: The **Fibonacci Sequence** is the sequence of numbers: 1, 1, 2, 3, 5, 8, 13, 21, ...

What is interesting about the Fibonacci sequence is that if you take the ratio of two successive terms, the number converges to \( \Phi \) as the terms that you are taking the ratio of get larger!
1/1 = 1
2/1 = 1
3/2 = 1.5
5/3 = 1.6666666...
8/5 = 1.6
13/8 = 1.625
21/13 = 1.615384615
:
:
2584/1597 = 1.618034 which is approximately Φ.

5. APPLICATIONS FOR INTEGRATED ENERGY TECHNOLOGY

In problems 129 – 132, answer the questions. Use the conversion factors: 1 ft³ = 1728 in³, and 1 ft³ = 7.4805 gal. Round each answer to the nearest hundredth.

129. A bucket in the shape of a right circular cylinder with radius 8 inches and height 24 inches takes 30 seconds to fill. Determine the flow rate in gallons per minute (gpm).

130. A bucket in the shape of a right circular cylinder with radius 7.5 inches and height 18 inches takes 15 seconds to fill. Determine the flow rate in gallons per minute (gpm).

131. A fish tank with dimensions 3 feet by 2 feet by 1 foot takes 2.5 minutes to fill. Determine the flow rate in gallons per minute (gpm).

132. A fish tank with dimensions 2.5 feet by 1.25 feet by 1.25 feet takes 1 minute 45 seconds to fill. Determine the flow rate in gallons per minute (gpm).

6. APPLICATIONS FOR PROCESS TECHNOLOGY

In addition to the geometric figures defined earlier in this module, we will add a few more geometric shapes and solids: (1) Ellipse, (2) Elliptical tank, (3) Frustum-shaped tank

**Ellipse**: An ellipse is the set of all points (x, y) the sum of whose distances from two distinct points (**foci**) that lie on the **major axis** (i.e., axis that divides the ellipse in two equal parts lengthwise) is constant. The **minor axis** is the shorter axis that divides the ellipse in two equal parts. In the figure below, the length of the major axis is 2a, and the length of the minor axis is 2b. Note: An ellipse is NOT a polygon.
<table>
<thead>
<tr>
<th>Plane Geometric Figure</th>
<th>Circumference (C)</th>
<th>Area (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ellipse</strong></td>
<td>$C = 2\pi \sqrt{\frac{a^2 + b^2}{2}}$</td>
<td>$A = \pi ab$</td>
</tr>
</tbody>
</table>

**Elliptical Tank:** An elliptical tank is a cylinder in which the bases are ellipses and they are perpendicular to the height.

**Frustum-Shaped Tank:** A cone or pyramid with the top cut off is called a frustum. Many tanks, vats, cookers, and similar vessels encountered in steamfitting work are shaped like frustum.

Note: To find the volume of a frustum-shaped tank, you need to calculate the area of each base. If the frustum has circular bases, then the area is $\pi r^2$ (for each base), and if the frustum has rectangular bases, then the area is length times width (for each base).
<table>
<thead>
<tr>
<th>Geometric Solid</th>
<th>Volume ((V))</th>
<th>Surface Area ((S))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Elliptical Tank</strong></td>
<td>( V = \pi abh )</td>
<td>( S = 2\pi \left(ab + h\sqrt{\frac{a^2 + b^2}{2}}\right) )</td>
</tr>
<tr>
<td>(a, b) = distance from center to vertices (h) = height</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Frustum-Shaped Tank</strong></td>
<td>( V = \frac{ah + bh + h\sqrt{ab}}{3} )</td>
<td>N/A</td>
</tr>
<tr>
<td>(a) = area of the upper base (b) = area of the lower base (h) = altitude, or distance between bases</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In problems 133 – 142, answer the questions. Round each answer to the nearest hundredth.

133. Find the area of an ellipse in which the major axis has a length of 8 feet and the minor axis has a length of 6 feet. Find the circumference of this same ellipse.

134. Find the volume of an elliptical tank in which the major axis has a length of 36 inches, the minor axis has a length of 24 inches, and the height is 120 inches. How many gallons of water will this tank hold (assume 1 ft\(^3\) = 7.4805 gal).

135. Find the surface area of the elliptical tank given in problem 134 (on the previous page).

136. Find the volume of a frustum-shaped tank with circular bases with radii 1.5 ft, and 0.8 ft, and a height of 0.75 ft.

137. Find the volume of a frustum-shaped tank with rectangular bases. The width and length of the lower base is 7 in. by 8 in., the width and length of the upper base is 4 in. by 4.57 in., and the height is 6 in.
138. Find the volume of the frustum-shaped tank in the figure below. Each measurement is given in feet.

![Frustum-shaped tank diagram]

139. Find the volume of the tank given in the photo below. The right circular cylindrical tank has a radius of approximately 1.5 feet and a height of approximately 3 feet.

![Cylindrical tank photo]

140. Find the surface area of the tank given in the photo below. The rectangular tank has a length of approximately 4.5 feet, width of 4 feet, and height of 4.5 feet.

![Rectangular tank photo]
141. Find the volume of the tank given in the photo below. You may assume that the tank is a right circular cylinder with a semi-sphere on both ends. The radius for the cylinder and semi-sphere is approximately 15 inches and the height (from end to end) is approximately 75 inches.

\[ V = \pi r^2 h + \frac{2}{3} \pi r^3 \]

\[ r = 15 \text{ in.} \]
\[ h = 75 \text{ in.} \]

142. Find the volume of the hydrocyclone given below ignoring the inlet and outlet cylinders. The cone has a height of 25 inches, the cylinder has a height of 20 inches and a radius of 5 inches.

\[ V = \frac{1}{3} \pi r^2 h + \pi r^2 h \]

7. APPLICATIONS FOR SKI AREA OPERATIONS

In terrain parks and half pipes, snow is typically angled off on each side of a jump at a 45° angle creating geometric solids such as triangular prisms and frustums. We have already defined frustums and their volume. We will now define the volume of a right triangular prism.

\[ V = \frac{1}{2}bh \times h \]
**Right Triangular Prism:** A right triangular prism is a geometrical solid with identical triangular bases. Notice that the triangular base has a right angle, therefore, properties of right triangles given in the previous module hold true here.

<table>
<thead>
<tr>
<th>Geometric Solid</th>
<th>Volume (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Right Triangular Prism</strong></td>
<td>$V = \frac{lwh}{2}$</td>
</tr>
<tr>
<td>$l = \text{length}$</td>
<td></td>
</tr>
<tr>
<td>$w = \text{width}$</td>
<td></td>
</tr>
<tr>
<td>$h = \text{height}$</td>
<td></td>
</tr>
<tr>
<td><strong>Frustum-Shaped Jump (Tabletop Jump)</strong></td>
<td>$V = \frac{ah + bh + h\sqrt{ab}}{3}$</td>
</tr>
<tr>
<td>$a = \text{area of the upper base}$</td>
<td></td>
</tr>
<tr>
<td>$b = \text{area of the lower base}$</td>
<td></td>
</tr>
<tr>
<td>$h = \text{altitude, or distance between bases}$</td>
<td></td>
</tr>
</tbody>
</table>

In problems 143 – 146, answer the questions. Round each answer to the nearest hundredth.

143. Below is a cross-sectional view of a half pipe. Find the volume of snow that is needed to construct this half pipe assuming that the length of the half pipe is 500 feet. Recall that the volume of a solid with identical bases is defined to be the area of the base times the height. The half pipe can be considered a rectangular solid with half of a right circular cylinder cut out of it.
144. Calculate the blade capacity of a dozer blade (right triangular prism) with the following dimensions: length = 8 feet, width = 5 feet, and height = 5 feet.

145. How many full scoops of a dozer with the blade capacity given in problem 143 would it take to completely level (tear down) the half pipe given in problem 142?

146. A tabletop jump (as in the photo given below) is in the shape of a frustum. How much snow would it take to make a tabletop jump in which the lower rectangular base measures 20 feet by 40 feet and the upper rectangular base measures 12 feet by 24 feet, and the height is 8 feet?

Solutions to Module VIII:
1. Quadrilateral or Trapezoid  2. Pentagon  3. Decagon  4. Heptagon  5. Hexagon  6. Octagon 7. True  8. False  9. False  10. True  11. True  12. No  13. Yes  14. No  15. Yes  16. Yes 17. No  18. 14.43 mm  19. 20.8 in.  20. 22 2/3 m  21. 23.4 ft  22. 25.73 m  23. 57 cm 24. 14.76 ft  25. 16 ½ yd  26. 28.28 cm; 40.57 cm²  27. 62.56 m; 224.81 m²  28. 19 in.; 19.35 in²  29. 100 cm²  30. 78.54 yd²  31. 20 in.  32. 15.59 ft²  33. 12 m²  34. 18 mm²  35. 55.94 m² 36. 289.5 ft  37. 1260 ft  38. $262.50  39. Approximately 36 ft  40. 69 times a day 41. 1051.35 km each week  42. $228418 \pi \text{ cm} + \pi \text{ cm} \quad \text{122146} \pi \quad \text{43.} \quad 2774 \pi \text{ mm} \quad \text{362} \pi \quad \text{44.} \quad \text{Because we don’t distinguish sides and bases of a sphere. The lateral area is the surface area of a sphere.} \quad 45. \quad 192 \text{ m}² \quad \text{46.} \quad 240 \text{ m}² \quad \text{47.} \quad 65.97 \text{ ft}² \quad \text{48.} \quad 113.10 \text{ m}² \quad 49. \quad 1256.64 \text{ m}² \quad \text{50.} \quad 658.8 \text{ ft}²; \quad 1010.8 \text{ ft}² \quad \text{51.} \quad 201.64 \text{ m}²; \quad 302.46 \text{ m}² \quad \text{52.} \quad 90.73 \text{ in}²; \quad 113.41 \text{ in}² \quad 53. \quad 207.35 \text{ ft}² \quad \text{54.} \quad 703.72 \text{ in}² \quad \text{55.} \quad 192 \text{ in}² \quad \text{56.} \quad 50.65 \text{ m}³ \quad \text{57.} \quad 59.61 \text{ ft}³ \quad \text{58.} \quad 14.14 \text{ in}³ \quad 59. \quad 1443.32 \text{ ft}³ \quad \text{60.} \quad 504 \text{ ft}³ \quad \text{61.} \quad 2304 \text{ ft}³ \quad \text{62.} \quad 1 \text{ to} 3 \quad \text{63.} \quad 125 \text{ m}³ \quad \text{64.} \quad 314.16 \text{ ft}³ \quad \text{65.} \quad 150.80 \text{ in}³ \quad 66. \quad 141.37 \text{ m}³ \quad \text{67.} \quad 392\pi \text{ m}³; \quad 224\pi \text{ m}² \quad \text{68.} \quad \text{a)} \quad 972\pi \text{ cm}³; \quad 324\pi \text{ cm}² \quad \text{69.} \quad \text{a)} \quad 4 \text{ in.} \quad \text{b) 96 in}³ \quad 70. \quad 256\pi \text{ m}³ \quad 71. \quad 160 - 18\pi \text{ in}³ \quad 72. \quad 896 \text{ in}³ \quad \text{73.} \quad 196\pi \text{ in}³ \quad \text{74.} \quad 154\pi \text{ in}² \quad \text{75.} \quad 56\pi \text{ in}² \quad 76. \quad 48 \text{ m}³; \quad 96 \text{ m}²; \quad 60 \text{ m}² \quad 75. \quad 168 \text{ ft}³; \quad 244 \text{ ft}²; \quad 196 \text{ ft}² \quad \text{76.} \quad \text{Sphere is} \quad 164.64 \text{ in}³ \quad \text{and the Cylinder is} \quad 180.96 \text{ in}³ \quad \text{however, it may be easier to drink out of the cylinder.} \quad \text{77.} \quad 175 \text{ ft} \quad \text{78.} \quad 5.5 \text{ gallons} \quad \text{79.} \quad \text{No; volume will increase by a factor of 27.} \quad \text{80.} \quad 560 \text{ ft}² \text{ of wallpaper} \quad \text{81.} \quad 672 \text{ ft}² \text{ of wallpaper} \quad 82. \quad 78.54 \text{ ft}³ \text{ of liquid} \quad \text{83.} \quad 24 \text{ ft}³ \quad \text{84.} \quad 0.15 \text{ mL} \quad \text{85.} \quad 0.24 \text{ mL} \quad \text{86.} \quad 0.32 \text{ mL} \quad \text{87.} \quad 0.44 \text{ mL} \quad \text{88.} \quad 0.55 \text{ mL} \quad \text{89.} \quad 1.01 \text{ mL} \quad \text{90.} \quad \text{Answers vary} \quad \text{91.} \quad 37 \text{ chains} \quad \text{92.} \quad \text{iii. 5683 sq paces} \quad \text{93.} \quad \text{ii. 5 acres} \quad \text{94.} \quad \text{Approximately 289.2 min. or 4 hours 49.2 min. 95.} \quad \text{Approximately 4.1 gallons} \quad \text{96.} \quad 9.2 \text{ gal} \quad \text{97.} \quad 4.1 \text{ gal} \quad \text{98.} \quad 1.2 \text{ gal} \quad \text{99.} \quad 0.8 \text{ gal} \quad \text{100.} \quad 55.2 \text{ gallons} \quad \text{101.} \quad 133 \text{ gallons} \quad \text{102.} \quad 4,150 \text{ lbs.} \quad \text{103.} \quad 206.7 \text{ lbs.} \quad \text{104.} \quad 3.62 \text{ fl-oz} \quad \text{105.} \quad 11.45 \text{ fl-oz} \quad \text{106.} \quad \text{Approximately 5.59 ice cream cones} \quad \text{107.} \quad 54.4 \text{ fl-oz} \quad \text{108.} \quad \text{Cube} = 24.39 \text{ ft}³, \text{Cylinder} = 25.12 \text{ ft}³, \text{Rectangle} = 25.33 \text{ ft}³,
therefore the rectangular carton has the greatest storage space. 109. Cube = 50.46 ft², Cylinder = 50.24 ft², Rectangle = 54.78 ft², therefore the rectangular carton will need the most wrapping paper. 110. 15 megapixels 111. 21 megapixels 112. 18 megapixels 113. 12 megapixels 114. 12 megapixels 115. 24 megapixels 116. 14.29 by 9.49 in. 117. 20.16 by 13.44 in. 118. 15.84 by 10.56 in. 119. 18.72 by 12.48 in.

<table>
<thead>
<tr>
<th>Megapixels</th>
<th>Pixel Resolution*</th>
<th>Print Size @ 300ppi</th>
<th>Print size @ 200ppi</th>
<th>Print size @ 150ppi**</th>
</tr>
</thead>
<tbody>
<tr>
<td>120. 3</td>
<td>2048 x 1536</td>
<td>6.82&quot; x 5.12&quot;</td>
<td>10.24&quot; x 7.68&quot;</td>
<td>13.65&quot; x 10.24&quot;</td>
</tr>
<tr>
<td>121. 4</td>
<td>2464 x 1632</td>
<td>8.21&quot; x 5.44&quot;</td>
<td>12.32&quot; x 8.16&quot;</td>
<td>16.42&quot; x 10.88&quot;</td>
</tr>
<tr>
<td>122. 6</td>
<td>3008 x 2000</td>
<td>10.02&quot; x 6.67&quot;</td>
<td>15.04&quot; x 10.00&quot;</td>
<td>20.05&quot; x 13.34&quot;</td>
</tr>
<tr>
<td>123. 8</td>
<td>3264 x 2448</td>
<td>10.88&quot; x 8.16&quot;</td>
<td>16.32&quot; x 12.24&quot;</td>
<td>21.76&quot; x 16.32&quot;</td>
</tr>
<tr>
<td>124. 10</td>
<td>3872 x 2592</td>
<td>12.91&quot; x 8.64&quot;</td>
<td>19.36&quot; x 12.96&quot;</td>
<td>25.81&quot; x 17.28&quot;</td>
</tr>
<tr>
<td>125. 12</td>
<td>4290 x 2800</td>
<td>14.30&quot; x 9.34&quot;</td>
<td>21.45&quot; x 14.00&quot;</td>
<td>28.60&quot; x 18.67&quot;</td>
</tr>
<tr>
<td>126. 16</td>
<td>4920 x 3264</td>
<td>16.40&quot; x 10.88&quot;</td>
<td>24.60&quot; x 16.32&quot;</td>
<td>32.80&quot; x 21.76&quot;</td>
</tr>
</tbody>
</table>

127. $L = \frac{1 + \sqrt{5}}{2} \approx 1.618033989$ 128. According to the Greeks, triangle C is the most pleasing to look at. 129. 41.78 gpm 130. 55.08 gpm 131. 17.95 gpm 132. 16.70 gpm 133. $A = 37.7 \text{ ft}^2$; $C = 22.21 \text{ ft}$ 134. $V = 81430.08 \text{ in}^3$; 349.12 gallons 135. 12890.88 in² 136. 3.21 ft³ 137. 212.55 in³ 138. 453.58 ft³ 139. 21.21 ft³ 140. 112.5 ft³ 141. 45945.79 in³ 142. 2225.29 in³ 143. 609867.29 ft³ 144. 100 ft³ 145. 6099 scoops 146. 4181.33 ft³
Module IX
Data Graphs and Coordinate Geometry

Graphing is a method of showing the relationship between two or more sets of data by means of a chart or sketch. With graphs it is easier for us to recognize trends in data that help us make predictions or conjectures about the relationship between the data. In this section we will identify and interpret four types of graphs: (1) Circle Graph or Pie Chart, (2) Bar Graph, (3) Line Graph, and (4) Graphs in the Rectangular Coordinate System.

I. Circle Graphs
Khan Academy Resources: https://www.khanacademy.org/math/pre-algebra/pre-algebra-math-reasoning/pre-algebra-picture-bar-graphs/v/reading-pie-graphs-circle-graphs

Suppose that in a Colorado Mountain College MAT 107 class of 20 students, 4 student received A’s, 5 students received B’s, 6 students received C’s, 3 students received D’s and 2 students received F’s. If we would like to illustrate the percentage of students receiving each letter grade we could use a circle graph (or pie chart). A circle graph separates each category and lists it as a percentage of a whole amount. Visually, each category is drawn in the shape of a “piece of pie” that corresponds to the percentage (or fraction) of the whole circle. For instance, if ½ or 50% of the students passed, in a pie chart, one-half of the circle would represent the number of students that passed.

Now we’ll create a circle graph for the MAT 107 class of 20 students that we described earlier. First, we need to find the percentages of students receiving each letter grade:

- 4 A’s = 4/20 = 20%
- 5 B’s = 5/20 = 25%
- 6 C’s = 6/20 = 30%
- 3 D’s = 3/20 = 15%
- 2 F’s = 2/20 = 10%

Notice that the sum of all of the percentages is 100%. If this is not the case, then most likely a mistake was made. Sometimes, with awkward percentages, rounding errors will cause the percentages to add incorrectly.

Now, with the percentages given above, we can assign a “slice of pie” for each category. See Figure 9.1 on the next page.
Example 1: If there were a total of 75 students in MAT 107, and the grade percentage was the same as in Figure 9.1 above, how many students received an A?

Solution to Example 1:
In Figure 9.1, we see that 20% of the total students received A’s. This means that we need to find 20% of 75. 20% of 75 = 0.2 \times 75 = 15. **Answer: 15 students received A’s**

II. Bar Graphs


Plotting **Bar graphs** is another way to organize and illustrate data. Usually, in a bar graph, each category is plotted by using the frequency in which it occurred. The frequency is illustrated with a rectangular region, or “bar”. That is, the higher the frequency, the higher the bar on the graph. For instance, in our MAT 107 class of 20 students, we would have the same five categories, but instead of plotting the percentage of each student receiving a given grade (as we did in a circle graph), in a bar graph we would plot the frequency: A’s = 4, B’s = 5, C’s = 6, D’s = 3, F’s = 2. The width of each bar bears no meaning, so it does not matter how wide your bar’s are. The bar graph in Figure 9.2 below plots the grade vs. frequency.
Example 2: Use the bar graph in Figure 9.2 above to determine how many students passed MAT 107, that is, received a C or better in the class.

Solution to Example 2:
Finding the categories “A’s”, “B’s”, and “C’s” we can determine the frequency of each. We see that there are 4 A’s, 5 B’s, and 6 C’s, therefore 15 students received a C or better.
Answer: 15 students passed MAT 107

III. Line Graphs

Line graphs illustrate the same data as bar graphs, except each point between consecutive categories is connected with a straight line. The line helps us visualize the slope between each category on the horizontal axis. When analyzing stock market trends, for example, it is important to know the rate in which the stock is increasing. A line graph will help illustrate this trend. The line graph in figure 9.3 on the next page plots the MAT 107 grades vs. frequency.

Example 3: Use the line graph on the next page to determine the largest amount of decrease in frequency from two consecutive grades.

Solution to Example 3:
Looking at the line graph in Figure 9.3 we see that there is the steepest drop in frequency from C’s to D’s. Answer: The largest amount of decrease in frequency is from C’s to D’s.
IV. Rectangular Coordinate System

Khan Academy Resources:


The Rectangular Coordinate System (or Cartesian Coordinate System), named after French mathematician and philosopher René Descartes) is a coordinate system that is used to graph lines in the plane. The Cartesian plane consists of two intersecting axes; the x-axis or horizontal axis, and the y-axis or vertical axis. The axes intersect in a point called the origin, in which it is assigned the ordered pair (0, 0). An ordered pair (or coordinate, or point) is a pair of numbers that are ordered so that the x-coordinate (or abscissa) is listed first, and the y-coordinate (or ordinate) is listed second to form the point (x, y). To plot ordered pairs in the plane we start at the origin, and then move a horizontal distance of x units (positive is to the right, and negative is to the left) and a vertical distance of y units (positive is up, and negative is down). There are four quadrants in the rectangular coordinate system. Quadrant I starts in the upper right portion, and then the following quadrants are listed in a counterclockwise manner. Figure 9.4 below illustrates the rectangular coordinate system.
Let us now define a few terms before we begin graphing lines in the plane.

**Slope-Intercept Form of a Line:** Any linear equation of the form \( y = mx + b \) is in slope-intercept form where \( m \) is the slope and \((0, b)\) is the \(y\)-intercept.

**x-intercept:** The point on the line that touches the \(x\)-axis. To find the \(x\)-intercept, let \( y = 0 \), then solve for \(x\). The \(x\)-intercept written as a coordinate is of the form \((a, 0)\).

**y-intercept:** The point on the line that touches the \(y\)-axis. To find the \(y\)-intercept, let \( x = 0 \), then find \(y\). The \(y\)-intercept written as a coordinate is of the form \((0, b)\).

**Slope:** Slope is a rate of increase (positive slope) or a rate of decrease (negative slope) in which the rate remains constant. If given two points on a line to be \((x_1, y_1)\) and \((x_2, y_2)\) the slope of the line is

\[
\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

**Special Forms of lines:**
1. A line with zero slope \((m = 0)\) yields a horizontal line parallel to the \(x\)-axis. The equation of this line is of the form \(y = b\), for some constant \(b\). The line \(y = b\) must pass through the point \((0, b)\).
2. A line with no slope (or undefined slope) yields a vertical line parallel to the \(y\)-axis. The equation of this line is of the form \(x = a\), for some constant \(a\). The line \(x = a\) must pass through the point \((a, 0)\).

**Graphing Lines in the Plane:**
Two points determine a line. If we can find two distinct points that satisfy the line we can then plot the points and draw a straight line through them. To find two distinct points, you may use the \(x\)- and \(y\)-intercept, or just randomly find two points by substituting an \(x\)-value into the linear equation and then finding its corresponding \(y\)-value. You may repeat this process to find as many points as you like. If the equation of the line is given to us in slope-intercept form, we
may then plot the y-intercept and then use the slope to find another point. Remember that slope is “rise over run”, so a slope of 2, for example, would tell us to go up 2 and to the right 1 from the lines y-intercept to get to another point on the line.

**Example 4:** Given the linear equation $3x - y = 6$;

a) Find the x-intercept and write your answer as a coordinate.

b) Find the y-intercept and write your answer as a coordinate.

c) Write the equation in slope-intercept form.

d) Find the slope of the line.

e) Graph the line on the Cartesian plane below.
Solution to Example 4:

a) To find the x-intercept we must let y = 0 in the equation 3x – y = 6, then solve for x.
   3x – 0 = 6 means that x = 2. So the x-intercept is (2, 0). Answer: (2, 0)

b) To find the y-intercept we must let x = 0 in the equation 3x – y = 6, then solve for y.
   3(0) – y = 6 means that y = -6. So the y-intercept is (0, -6). Answer: (0, -6)

c) To write 3x – y = 6 in slope-intercept form, we must solve the equation for y, and write it in
   the form y = mx + b. Solving for y, we get y = 3x – 6. Notice that this confirms that our y-
   intercept is (0, -6). Answer: y = 3x – 6

d) Since the line is in slope-intercept form from part (c), we can find the slope m: m = 3.
   Answer: The slope is 3

e) Since we already have two points found in (a) and (b) (the x- and y-intercepts), we can use
   these points to plot our line in the Cartesian plane

V. Linear Regression

Khan Academy Resources: https://www.khanacademy.org/math/statistics-probability/describing-
relationships-quantitative-data/regression-library/v/fitting-a-line-to-data

Now that we have investigated the fundamentals of linear equations in two variables, we will
apply our knowledge to linear regression models. Suppose that we would like to investigate the
correlation between Accuplacer Scores and Passage Rates for those students placing in MAT 121
(College Algebra) via Accuplacer. Ultimately, we want to use the data to help us predict (with
some level of certainty) the chances for a student passing MAT 121, given their Accuplacer score.
Our graph may look like the graph given on the next page.
The scatter diagram (or scatterplot) above can be interpreted in many ways, that is, if we agree that the data appears to be following a linear pattern, increasing from left to right, then there are many different lines that may represent the data sufficiently (See Figure 9.5 below).

Even though the two lines drawn in Figure 9.5 are good estimates, there is only one line, however, that BEST fits the data. That line is called the least squares regression line. The regression line is the line for which the sum of the squares of the vertical distances between the data points and the line is a minimum. A graphing calculator can compute the least squares regression line given any amount of data points rather easily. In this section we will be estimating the least squares regression line without the use of a calculator.
**Example 5:** Suppose that the green line in Figure 9.5 is the “line of best fit” in which the line passes through the two “outer” points: (85, 45) and (115, 95)

a) Find the equation of the line of best fit.

b) Use the equation found in (b) to predict the passage rate for someone scoring 94 on the Accuplacer.

**Solution to Example 5:**

a) To find the equation of the line passing through the points (85, 45) and (115, 95), we will first need to find the slope. The slope formula is \( m = \frac{y_2 - y_1}{x_2 - x_1} \). Substituting, we get

\[
m = \frac{95 - 45}{115 - 85} = \frac{5}{3}.
\]

Now we need to find the y-intercept. Substituting the point (85, 45) and the slope \( m = \frac{5}{3} \) into the equation \( y = mx + b \) we get

\[
45 = \frac{5}{3}(85) + b.
\]

Solving for \( b \), we get

\[
b = -\frac{290}{3}.
\]

Our equation now becomes \( y = \frac{5}{3}x - \frac{290}{3} \). **Answer:** \( y = \frac{5}{3}x - \frac{290}{3} \).

b) Using our equation with \( x = 94 \), we substitute and get

\[
y = \frac{5}{3}(94) - \frac{290}{3} = 60.
\]

**Answer:** 60% of students scoring 94 on the Accuplacer will pass MAT 121.

**Homework Set:**

In problems 1 – 4, use the given information to construct the appropriate graph.

1. The following is a list of deductions from Justin’s monthly pay. Construct a circle graph indicating each deduction as a percent of the total deductions. Round each value to the nearest percent.

<table>
<thead>
<tr>
<th>Deductions</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medicare</td>
<td>$99.98</td>
</tr>
<tr>
<td>Federal Tax</td>
<td>$507.83</td>
</tr>
<tr>
<td>State Tax</td>
<td>$207.00</td>
</tr>
<tr>
<td>Retirement</td>
<td>$540.34</td>
</tr>
<tr>
<td>Health Plan</td>
<td>$140.68</td>
</tr>
</tbody>
</table>

2. In a CMC survey, students were asked to respond to the statement: “Smoking should not be allowed on campus.” Students had the option to strongly agree, agree, be neutral, disagree, or strongly disagree. Construct a circle graph for the students response as a percent of the total students surveyed. Round each value to the nearest percent.

- Strongly agree – 68
- Agree – 198
- Neutral – 25
- Disagree – 94
- Strongly Disagree – 35

3. In the table below, Jenny’s average daily energy usage is given in kWh (kilowatt-hours), for the months of January through June, in the years 2009 and 2010. Construct a vertical bar graph with the month along the horizontal axis and the average daily energy use on the vertical axis. Show year 2009 and year 2010 as separate bars, side-by-side for each month.

<table>
<thead>
<tr>
<th>Month</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>February</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>March</td>
<td>21</td>
<td>30</td>
</tr>
<tr>
<td>April</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>May</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>June</td>
<td>19</td>
<td>15</td>
</tr>
</tbody>
</table>

4. Construct a line graph for the data in problem (3) above. Show year 2009 and year 2010 as separate lines.

In problems 5 – 10, find (a) the x-intercept if any, (b) the y-intercept if any, (c) the slope of the line, and (d) graph the line.

5. \( y = -\frac{1}{2}x - 4 \)
6. \( y = 3x + 8 \)
7. \( 5x - 4y = 20 \)
8. \( 7x - 3y = 6 \)
9. \( y = -2 \)
10. \( x = 7 \)
In problems 11 – 18, use the given information to find the slope of the line.

11. Find the slope of the line \( y = -2x - 4 \).
12. Find the slope of the line \( y = \frac{3}{4}x + \frac{1}{2} \).
13. Find the slope of the line containing the points \((0, 5)\) and \((7, 8)\).
14. Find the slope of the line containing the points \((-1, 3)\) and \((-4, 5)\).
15. Find the slope of the line containing the points \((6, 7)\) and \((8, 7)\).
16. Find the slope of the line containing the points \((-4, -3)\) and \((-4, 7)\).
17. Suppose a car depreciates linearly the second you drive it off the lot. If you purchased the car for $26,500 and after 4 years the car is worth $18,700, find the slope of the depreciation line and explain its meaning in the context of the problem.
18. Suppose that the coyote population in Garfield County, Colorado grows at a constant rate (linear). If in 2007, the coyote population in Garfield County was 125 and in 2010, the coyote population increased to 175, find the slope of the Coyote growth line and explain its meaning in the context of the problem.

In problems 19 – 20, answer the questions.

19. Suppose a car depreciates linearly the second you drive it off the lot. You purchased the car for $26,500 and after 4 years the car is worth $18,700.
   a) Write a linear equation for the value, \( V \), of the car after, \( t \), years.
   b) Use the linear equation found in (a) to determine the value of the car after 7 years.
   c) Graph your equation on a Cartesian plane.
   d) Use your graph to determine how many years it will take the value of the car to reach $0.
20. Suppose that the coyote population in Garfield County, Colorado grows at a constant rate (linear). In 2007, the coyote population in Garfield County was 125 and in 2010, the coyote population increased to 175.
   a) Write a linear equation for the population, \( P \), of coyotes after, \( t \), years.
   b) Use the linear equation found in (a) to determine the population of coyotes in 2015.
   c) Graph your equation on a Cartesian plane.
   d) Use your graph to determine how many years it will take the coyote population to reach 375.
1. APPLICATIONS FOR EMT/MEDICAL ASSISTANT/NURSING

In problems 21 – 24, the half-life formula $y = 500(0.5)^{t/6}$ is given in the graph below in which $y$ is the amount of drug left in the bloodstream after $t$ hours from an initial amount of 500 mg of drug. Use the graph to answer the questions.

21. Use the graph above to approximate how much drug is left in the bloodstream 16 hours after a 500 milligram dose?
22. Use the graph above to approximate how much drug is left in the bloodstream 8 hours after a 500 milligram dose?
23. Use the graph above to approximate how long it will take a 500 mg dose to reach 150 mg in the bloodstream.
24. Use the graph above to approximate how long it will take a 500 mg dose to reach 25 mg in the bloodstream.

2. APPLICATIONS FOR FIRE SCIENCE

25. On a Cartesian plane, draw a graph for pump performance showing the relationship between pressure (psi) and flow (gpm) by using the table of pump performance data values given below. Let gpm be the horizontal axis and psi the vertical axis. Note: Your graph will NOT be linear!

<table>
<thead>
<tr>
<th>psi</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
<th>225</th>
<th>250</th>
<th>275</th>
<th>295</th>
</tr>
</thead>
<tbody>
<tr>
<td>gpm</td>
<td>79</td>
<td>77</td>
<td>75</td>
<td>69.5</td>
<td>63</td>
<td>56.5</td>
<td>51</td>
<td>46</td>
<td>40.5</td>
<td>29.5</td>
<td>0</td>
</tr>
</tbody>
</table>
In problems 26 – 29, use the graph below to answer the questions.

![Pump Performance Curve](image)

26. Find the pressure for a flow rate of 40 gpm.
27. Find the pressure for a flow rate of 75 gpm.
28. Find the flow rate at a pressure of 200 psi.
29. Find the flow rate at a pressure of 263 psi.
30. Find the slope of the line drawn on the graph above for the interval between 50 and 150 pounds per square inch. Explain the answers meaning in the context of the problem.

31. Use the mileage chart below to find the distance between Tampa, FL, and Albuquerque, NM.

<table>
<thead>
<tr>
<th>Mileage Chart, miles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Los Angeles, CA</td>
</tr>
<tr>
<td>Portland, OR</td>
</tr>
<tr>
<td>Albuquerque, NM</td>
</tr>
<tr>
<td>Denver, CO</td>
</tr>
</tbody>
</table>

FUN FACT: A **contour line** is a line drawn on a **topographic map** to indicate ground elevation or depression. A contour interval is the vertical distance or difference in elevation between contour lines. Index contours are bold or thicker lines that appear at every fifth contour line.
If the numbers associated with specific contour lines are increasing, the elevation of the terrain is also increasing. If the numbers associated with the contour lines are decreasing, there is a decrease in elevation. As a contour approaches a stream, canyon, or drainage area, the contour lines turn upstream. They then cross the stream and turn back along the opposite bank of the stream forming a "v". A rounded contour indicates a flatter or wider drainage or spur. Contour lines tend to enclose the smallest areas on ridge tops, which are often narrow or very limited in spatial extent. Sharp contour points indicate pointed ridges. Widely separated contour lines indicate a gentle slope. Contour lines that are very close together indicate a steep slope. Contour lines that are equally spaced indicates that the hill has a uniform slope. To determine a vertical change, pick two contour lines next to each other and find the difference in the numbers.
The figure above illustrates various topographic features. (b) Notice how a mountain saddle, a ridge, a stream, a steep area, and a flat area are shown with contour lines.

The figure above illustrates a depression and its representation using contour lines. Notice the tick marks pointing toward lower elevation.
32. In the graphic below, what is the vertical distance between the contour lines?

33. Draw a profile showing the elevations of the contours in the figure below. Note: The intervals are increasing, therefore, the contours indicate a hill. The peak is normally considered to be located at half the interval distance.

34. Use the figure to answer the following questions:
   a) Is the figure shown here steeper at the top of the hill or the bottom?

   b) On this topographic map, the contour interval is not indicated. What contour interval can you determine based on the available information?
3. APPLICATIONS FOR ACCOUNTING

35. Suppose that you open a business in which you realize that your monthly costs \( C \) (in thousands) can be modeled by the linear function \( C = \frac{1}{2}x + 4 \), and your businesses monthly revenue \( R \) (in thousands) can be modeled by the linear function \( R = \frac{3}{2}x \) where \( x \) is the number of units that you produce and sell for both models.
   a) Graph the cost and revenue functions on a Cartesian plane.
   b) Determine from your graph how many units in a month that you must sell in order to break-even, then confirm this amount algebraically.
   c) Use your cost and revenue functions to write a profit equation for your business. Remember, Profit = Revenue – Cost.
   d) Find the slope of your profit function and briefly describe what this value means in the context of your business.

36. Suppose that you are analyzing costs on your newly opened business in which you plan to sell hand-made flies for fishing. The materials and labor to make one fly is $0.35. You plan to sell each fly at a price of $2.45 each. If \( x \) represents the number of flies produced and sold, and \( R \) represents the revenue:
   a) Write a linear equation for the revenue in terms of the number of flies.
   b) Write a linear equation for the cost in terms of the number of flies.
   c) Use the equations found in (a) and (b) to write the profit equation. Remember, Profit = Revenue – Cost.
   d) If you produced and sold 35 flies, how much would you profit?
   e) Graph your profit equation on a Cartesian plane.
   f) Use your graph above to find how many flies that you must sell to profit $105.

4. APPLICATIONS FOR CULINARY ARTS

37. A restaurant has fixed costs of $150,000 and variable costs of 64% of the check average. The check average is $15.
   a) Write a linear equation that describes the revenue, \( R \), the restaurant receives from each customer, \( x \).
   b) Write a linear equation that describes the restaurants costs, \( C \), in terms of the number of each customer, \( x \).
   c) Write an equation for the restaurants profit, \( P \), in terms of each customer, \( x \).
   d) Graph your profit equation on a Cartesian plane.
   e) Use your graph in (d) to determine how many customers are required to break-even?
5. APPLICATIONS FOR GRAPHIC DESIGN/PROFESSIONAL PHOTOGRAPHY

38. A Graphics Communications Company had a gross income of $325,000 for the past year. The company’s accountant categorized the income dollars into five areas which included labor, materials, overhead, management, and profit as given below. Construct a circle graph for each category as a percent of the gross income.

Labor: $130,000
Materials: $65,000
Overhead: $48,750
Management: $32,500
Profit: Remainder of $

6. APPLICATIONS FOR INTEGRATED ENERGY TECHNOLOGY

FUN FACT: The graph below is a sun path chart for Gypsum, Colorado. In the chart we can determine the solar elevation (angle of elevation from the horizontal to the sun). The vertical axis represents the solar elevation and the horizontal axis represents the solar azimuth (the direction of the sun in the horizontal plain from a given location).
In the sun path chart on the previous page, the blue bell-shaped curves represent each month (the top curve being June, and the bottom curve being December), and the red lines represent the time of day. For example, on December 21st at noon, the solar elevation is approximately 27° in Gypsum, Colorado.

**In problems 39 – 46, use the sun path chart for Gypsum, Colorado to answer the questions.**

39. Find the solar elevation for August 22nd at 6 pm in Gypsum, Colorado.
40. Find the solar elevation for December 21st at 11 am in Gypsum, Colorado.
41. Find the solar elevation for September 22nd at 10 am in Gypsum, Colorado.
42. Find the solar elevation for October 21st at 2 pm in Gypsum, Colorado.
43. Find the solar elevation for November 21st at 8 pm in Gypsum, Colorado.
44. Find the solar elevation for July 21st at 1 pm in Gypsum, Colorado.
45. At what date and time in Gypsum Colorado is the solar elevation the highest?
46. What month of the year in Gypsum Colorado produces the least amount of solar exposure?
47. Go to the website [http://solardat.uoregon.edu/SunChartProgram.php](http://solardat.uoregon.edu/SunChartProgram.php) and create a sun path chart for where you live.

**7. APPLICATIONS FOR PROCESS TECHNOLOGY**

**FUN FACT:** Many of the gauge instruments used to measure fluid levels in process technology equipment take a reading from a volume in a tank and produce an output in milliamps (mA), volts (V), or pounds per square inch (PSI). That is, the gauge reads the volume of a tank in terms of what percent of the tank is full, then the pneumatic signal system converts this percent to milliamps, volts, or pounds per square inch (depending on what type of gauge). We can use linear equations to help us convert from percentages to milliamps, or volts, or pounds per square inch. The following exercises address these conversions.

**In problems 48 – 50, answer the questions. Round each answer to the nearest hundredth, if necessary.**

48. If a tank is 100% full, then the digital signal on a gauge reads 20 mA. If the tank is 0% full, then the digital signal reads 4 mA.
   a) Use this information to describe two ordered pairs \((x, y)\) where \(x\) is the tank percentage and \(y\) is the milliamps reading.
   b) Use the points described in part (a) to find the slope of the line between the two points.
   c) Write an equation in slope-intercept form relating the percentage to the milliamps.
   d) Use your equation in (c) to determine what the gauge will read (in mA) of a tank that is 70% full.
   e) Use your equation in (c) to determine what percent of the tank is full given that the gauge reads 4.7 mA.
49. If a tank is 100% full, then the analog signal on a gauge reads 15 PSI. If the tank is 0% full, then the digital signal reads 3 PSI.
   a) Use this information to describe two ordered pairs \((x, y)\) where \(x\) is the tank percentage and \(y\) is the pounds per square inch reading.
   b) Use the points described in part (a) to find the slope of the line between the two points.
   c) Write an equation in slope-intercept form relating the percentage to the PSI.
   d) Use your equation in (c) to determine what the gauge will read (in PSI) of a tank that is 45% full.
   e) Use your equation in (c) to determine what percent of the tank is full given that the gauge reads 12.7 PSI.

50. If a tank is 100% full, then the digital signal on a gauge reads 5 V. If the tank is 50% full, then the digital signal reads 3 V.
   a) Use this information to describe two ordered pairs \((x, y)\) where \(x\) is the tank percentage and \(y\) is the volts reading.
   b) Use the points described in part (a) to find the slope of the line between the two points.
   c) Write an equation in slope-intercept form relating the percentage to the volts.
   d) Use your equation in (c) to determine what the gauge will read (in V) of a tank that is 92% full.
   e) Use your equation in (c) to determine what percent of the tank is full given that the gauge reads 2.2 V.
   f) A 42 gallon tank is filled with 10 gallons of fluid. What is the digital signal, in volts, of this tank?

8. APPLICATIONS FOR SKI AREA OPERATIONS

In problems 51 – 52, answer the questions. Round each answer to the nearest hundredth, if necessary.

51. To dig a snowmaking trench you plan to rent a front loader for 7 days and hire an operator to complete the job. It costs $250/day to rent the front loader and you plan on paying an operator $18/hour to run the machine.
   a) Write a linear equation in slope-intercept form describing the total cost to complete the job, \(y\), in terms of the number of hours, \(x\), that the operator works.
   b) Use your equation found in part (a) to determine the total cost to complete the job if you need to hire an operator for 32 hours.
   c) Use your equation found in part (a) to determine how many hours an operator was hired if the total cost of the job is $2182.
52. To dig a snowmaking trench you plan to rent a front loader for 5 days and hire an operator to complete the job. It costs $294/day to rent the front loader and you plan on paying an operator $21.50/hour to run the machine.

a) Write a linear equation in slope-intercept form describing the total cost to complete the job, \( y \), in terms of the number of hours, \( x \), that the operator works.

b) Use your equation found in part (a) to determine the total cost to complete the job if you need to hire an operator for 22 hours.

c) Use your equation found in part (a) to determine how many hours an operator was hired if the total cost of the job is $2308.50.

Solutions to Module IX:

1. Medicare Federal Tax State Tax Retirement Health Plan

![Pie chart showing percentages of Medicare, Federal Tax, State Tax, Retirement, and Health Plan]

2. Strongly Agree Agree Neutral Disagree Strongly Disagree

![Pie chart showing percentages of Strongly Agree, Agree, Neutral, Disagree, Strongly Disagree]
5. \((-8, 0); (0, -4); -\frac{1}{2}\) 
6. \((-\frac{8}{3}, 0); (0, 8); 3\) 
7. \((4, 0); (0, -5); \frac{5}{4}\) 
8. \((\frac{6}{7}, 0); (0, -2); \frac{7}{3}\) 
9. None; \((0, -2); 0\) 
10. \((7, 0);\) None; Undefined slope; Vertical line

3.

4.

5. \((-8, 0); (0, -4); -\frac{1}{2}\) 
6. \((-\frac{8}{3}, 0); (0, 8); 3\) 
7. \((4, 0); (0, -5); \frac{5}{4}\) 
8. \((\frac{6}{7}, 0); (0, -2); \frac{7}{3}\) 
9. None; \((0, -2); 0\) 
10. \((7, 0);\) None; Undefined slope; Vertical line

11. \(-2\) 
12. \(-\frac{4}{3}\) 
13. \(3/7\) 
14. \(-2/3\) 
15. 0 
16. Undefined 
17. \(-1950\); The car depreciates $1950 every year 
18. \(50/3\); The coyote population in Garfield County increases by 50 every 3 years 
19. a) \(V = -1950 \cdot t + 26500\) 
   b) \$12,850 
   c) below 
   d) Approximately 13.6 years
20. a) \( P = \frac{50}{3} t + 125 \)  
b) Approximately 258 coyotes  
c) below  
d) 15 years

21. 75 mg  
22. 200 mg  
23. 10 hours  
24. 26 hours

25. **Pump Performance Curve**

26. 250 psi  
27. 100 psi  
28. 50 gpm  
29. 35 gpm  
30. -6 psi/gpm; The pressure is dropping 6 psi for every gpm  
31. 1760 miles  
32. 20 feet
33.

34. a) At the top, since the contour lines are closer together at the top. b) 40 ft.

35. a) Graph below  b) 4 units  c) $P = x - 4$  d) 1; For each unit sold, the profit increases by $1000

36. a) $R = 2.45x$  b) $C = 0.35x$  c) $P = 2.1x$  d) $73.50$  e) Graph below  f) 50 flies

37. a) $R = 15x$  b) $C = 9.6x + 150000$  c) $P = 5.4x - 150000$  d) Graph below  e) Approximately 27,778 customers
39. 10° 40. 25° 41. 42° 42. 32° 43. 0° 44. 68° 45. June 21st at approximately 12 pm 46. December 47. You do this. 48. a) (100, 20), (0, 4) b) 4/25 c) $y = (4/25)x + 4$ d) 15.2 mA e) 4.38% 49. a) (100, 15), (0, 3) b) 3/25 c) $y = (3/25)x + 3$ d) 8.4 PSI e) 80.83% 50. a) (100, 5), (50, 3) b) 1/25 c) $y = (1/25)x + 1$ d) 4.68 V e) 30% f) 1.95 V 51. a) $y = 18x + 1750$ b) $2326$ c) 24 hours 52. a) $y = 21.5x + 1470$ b) $1943$ c) 39 hours